

Miscellaneous Mathematics Formulas

This cheat sheet covers a robust collection of miscellaneous mathematical identities, series expansions, constants, and vector calculus relations frequently required in advanced examinations.

1. Notable Constants

- Euler's Number: $e \approx 2.71828$
- Pi: $\pi \approx 3.14159$
- Euler-Mascheroni Constant: $\gamma \approx 0.57721$
- Golden Ratio: $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61803$

2. Series Expansions

Maclaurin series for common functions around $x = 0$:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots \quad (|x| < 1)$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

3. Gamma and Beta Functions

Gamma Function $\Gamma(z)$:

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$$

- $\Gamma(n) = (n-1)!$ for positive integer n .
- $\Gamma(1/2) = \sqrt{\pi}$

Beta Function $B(x, y)$:

$$B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

4. Vector Calculus Identities

Let ϕ be a scalar field and $\mathbf{A}, \mathbf{B}$ vector fields.

- $\nabla \cdot (\nabla \times \mathbf{A}) = 0$
- $\nabla \times (\nabla \phi) = \mathbf{0}$
- $\nabla \cdot (\phi \mathbf{A}) = (\nabla \phi) \cdot \mathbf{A} + \phi (\nabla \cdot \mathbf{A})$
- $\nabla \times (\phi \mathbf{A}) = (\nabla \phi) \times \mathbf{A} + \phi (\nabla \times \mathbf{A})$

5. Integrals of Interest

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \quad (a > 0)$$

$$\int_0^{\infty} \frac{\sin(x)}{x} dx = \frac{\pi}{2}$$