

Ultimate Algebra Formula Sheet

Elite Reference Guide for Advanced Mathematics

1. Exponents & Logarithms

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^k) = k \log_b x$$

$$\log_b x = \frac{\log_c x}{\log_c b}$$

2. Quadratic Equations

For $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant $\Delta = b^2 - 4ac$:

- $\Delta > 0 \implies$ 2 real roots
- $\Delta = 0 \implies$ 1 repeated root
- $\Delta < 0 \implies$ 2 complex roots

3. Polynomials & Binomials

Binomial Theorem:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. **Factoring Specials:**

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

4. Sequences & Series

Arithmetic Sequence:

$$a_n = a_1 + (n - 1)d$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

Geometric Sequence:

$$a_n = a_1 r^{n-1}$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}, \quad r \neq 1$$

5. Complex Numbers

Let $z = a + bi = r(\cos \theta + i \sin \theta) = re^{i\theta}$

$$|z| = \sqrt{a^2 + b^2}$$

$$\bar{z} = a - bi$$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

De Moivre's Theorem:

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

6. Matrices & Determinants

For 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$\det(A) = ad - bc$$

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

7. Inequalities & Absolute Value

$$|x| < c \implies -c < x < c$$

$$|x| > c \implies x < -c \text{ or } x > c$$

$$\text{AM-GM: } \frac{a+b}{2} \geq \sqrt{ab} \quad (a, b \geq 0)$$