Vectors and 3D Geometry Formula Sheet

Comprehensive Reference Guide for Advanced Mathematics

1. Vector Algebra Fundamentals

Position Vector: For point $P(x, y, z)$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Magnitude (Length): $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Unit Vector: $\hat{a} = \frac{\vec{a}}{|\vec{a}|}, \quad \vec{a} \neq \vec{0}$

Direction Cosines (l, m, n): $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

$$l^2 + m^2 + n^2 = 1$$

Section Formula: Position vector of point dividing line segment joining $\vec{a}$ and $\vec{b}$ in ratio $m : n$:

$$\vec{r} = \frac{m\vec{b} \pm n\vec{a}}{m \pm n}$$

(Plus for internal, minus for external division).

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2. Vector Products

Scalar (Dot) Product:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = a_1 b_1 + a_2 b_2 + a_3 b_3$$

* Angle between vectors:

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

— * Projection of $\vec{a}$ on $\vec{b}$:

$$\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

Vector (Cross) Product:

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

* Area of parallelogram $= |\vec{a} \times \vec{b}|$. * Area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$.

Scalar Triple Product:

$$[\vec{a}\vec{b}\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$$

* Represents volume of parallelpiped formed by $\vec{a}, \vec{b}, \vec{c}$.

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3. Lines in 3D Space

Vector Equation: Passing through point with position vector $\vec{a}$ and parallel to $\vec{b}$:

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

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Cartesian Equation:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

— where (x_1, y_1, z_1) is a point on the line and $\langle a, b, c\rangle$ are direction ratios.

Angle Between Two Lines: With direction ratios $\langle a_1, b_1, c_1\rangle$ and $\langle a_2, b_2, c_2\rangle$:

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Shortest Distance Between Skew Lines:

$$\text{SD} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

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4. Planes in 3D Space

Equation of a Plane: * Normal form: $\vec{r} \cdot \hat{n} = d$ * Cartesian form: $Ax + By + Cz + D = 0$ * Passing through point $\vec{a}$ with normal $\vec{n}$:

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

* Passing through three non-collinear points $\vec{a}, \vec{b}, \vec{c}$:

$$[(\vec{r} - \vec{a})\,(\vec{b} - \vec{a})\,(\vec{c} - \vec{a})] = 0$$

Angle Between Two Planes: Normal vectors $\vec{n}_1$ and $\vec{n}_2$:

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

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Distance from Point to Plane: Distance from (x_1, y_1, z_1) to $Ax + By + Cz + D = 0$:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$