

Arithmetic: Master Cheat Sheet

Comprehensive formulas, properties, and shortcuts for competitive examinations

1. Number Systems & Properties

Classification of Numbers:

- **Natural (N):** $\{1, 2, 3, \dots\}$
- **Whole (W):** $\{0, 1, 2, 3, \dots\}$
- **Integers (Z):** $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- **Rational (Q):** $\frac{p}{q}$ where $q \neq 0$
- **Irrational:** Non-terminating, non-repeating (e.g., $\pi, \sqrt{2}$)

Divisibility Rules:

- **3/9:** Sum of digits is divisible by 3/9.
- **4:** Last 2 digits are divisible by 4.
- **8:** Last 3 digits are divisible by 8.
- **11:** Diff. of alternate sum of digits is 0 or multiple of 11.

2. LCM & HCF

Fundamental Relations:

$$\text{HCF} \times \text{LCM} = \text{Product of two numbers}$$

$$\text{HCF of Fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

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Key Shortcuts:

- If H is the HCF of two numbers, they can be written as Hx and Hy where $\text{gcd}(x, y) = 1$.
- Greatest number dividing x, y, z leaving remainders a, b, c :

$$\text{HCF}(x - a, y - b, z - c)$$

3. Percentages

Core Conversions:

$$1/2 = 50\% \qquad 1/3 = 33.33\%$$

$$1/4 = 25\% \qquad 1/5 = 20\%$$

$$1/8 = 12.5\% \qquad 1/16 = 6.25\%$$

Percentage Change:

$$\% \text{ Change} = \frac{\text{Final} - \text{Initial}}{\text{Initial}} \times 100$$

Successive Percentage Change: If a value changes by $a\%$ and then by $b\%$, the net percentage change is given by:

$$\text{Net \%} = \left(a + b + \frac{ab}{100} \% \right)$$

4. Ratio & Proportion

Properties:

- **Duplicate ratio of $a : b$:** $a^2 : b^2$
- **Sub-duplicate:** $\sqrt{a} : \sqrt{b}$
- **Triplicate:** $a^3 : b^3$
- **Componendo & Dividendo:**

$$\text{If } \frac{a}{b} = \frac{c}{d} \implies \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Partnership Sharing:

$$\text{Profit Ratio} = \text{Investment} \times \text{Time Period}$$

5. Average & Mixture

Basic Formulas:

$$\text{Average} = \frac{\text{Sum of Observations}}{\text{Number of Observations}}$$

Weighted Average:

$$\text{Avg}_{mix} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Alligation Rule (Mixtures): Ratio of quantities of cheaper and dearer ingredients:

$$\frac{\text{Qty}_c}{\text{Qty}_d} = \frac{\text{Value}_d - \text{Avg}}{\text{Avg} - \text{Value}_c}$$

6. Profit

Core Formulas:

$$\text{Profit/Loss\%} = \frac{\text{Profit/Loss}}{\text{Cost Price (CP)}} \times 100$$

$$\text{SP} = \text{CP} \times \left(1 \pm \frac{\text{Profit/Loss\%}}{100} \right)$$

Marked Price & Discount:

$$\text{Discount} = \text{Marked Price (MP)} - \text{Selling Price (SP)}$$

$$\text{Discount\%} = \frac{\text{Discount}}{\text{MP}} \times 100$$

Successive Discounts (d_1, d_2):

$$\text{Equivalent Discount} = \left(d_1 + d_2 - \frac{d_1 d_2}{100} \% \right)$$

7. Interest Calculations**Simple Interest (SI):**

$$SI = \frac{P \times R \times T}{100}$$

Compound Interest (CI):

$$A = P \left(1 + \frac{R}{100} \right)^n$$

$$CI = A - P = P \left[\left(1 + \frac{R}{100} \right)^n - 1 \right]$$

Important Shortcuts:

- For 2 years, $CI - SI = \frac{PR^2}{100^2}$
- For 3 years, $CI - SI = \frac{PR^2(300+R)}{100^3}$

9. Time**Core Principles:**

- If a person does a piece of work in n days, work done in 1 day = $\frac{1}{n}$.
- Efficiency is **inversely proportional** to time taken.

Chain Rule Formula:

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$

Pipes and Cisterns:

- Inlet pipe = positive work (+)
- Outlet/Leak pipe = negative work (-)

8. Time**Fundamental Relation:**

$$\text{Distance} = \text{Speed} \times \text{Time}$$

Conversions:

$$1 \text{ km/hr} = \frac{5}{18} \text{ m/s}$$

Average Speed:

$$\text{Avg Speed} = \frac{2xy}{x+y} \quad (\text{for equal distances})$$

Relative Speed:

- Same direction: $S_1 - S_2$
- Opposite direction: $S_1 + S_2$