Masterclass Lesson Summary

Advanced Logical Reasoning

1 Introduction to Propositional Logic

Propositional logic forms the foundational bedrock of rigorous mathematical reasoning and analytical problem-solving. A proposition is a declarative statement that is either strictly True (T) or False (F), but never both.

Let P and Q be propositional variables. The primary logical connectives are defined as follows:

- **Conjunction (AND):** $P \land Q$ is true only when both P and Q are true.
- **Disjunction (OR):** $P \lor Q$ is true when at least one proposition is true.
- **Implication (Conditional):** $P \implies Q$ represents "If P, then Q." It is false only when P is true and Q is false.
- **Biconditional:** $P \iff Q$ means "P if and only if Q."

2 Truth Tables and Logical Equivalence

To analyze the truth value of complex compound statements, we construct truth tables. Two propositions A and B are logically equivalent, denoted $A \equiv B$, if they share identical truth values under every possible interpretation.

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Theorem 1 (De Morgan's Laws)**.** *For any propositions P and Q:*

$$\neg(P \land Q) \equiv \neg P \lor \neg Q$$
$$\neg(P \lor Q) \equiv \neg P \land \neg Q$$

3 Syllogisms and Deductive Reasoning

Deductive reasoning moves from general premises to a guaranteed specific conclusion. Consider the classic categorical syllogism:

1. **Premise 1:** All humans are mortal. ($\forall x, H(x) \implies M(x)$)
2. **Premise 2:** Socrates is a human. ($H(\text{Socrates})$)
3. **Conclusion:** Therefore, Socrates is mortal. ($M(\text{Socrates})$)

This form of valid inference is known as *Modus Ponens*:

$$\frac{P \quad P \implies Q}{Q}$$

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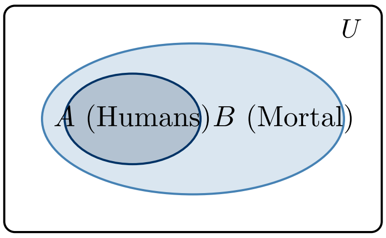
4 Visualizing Logic with Venn Diagrams

Set theory and logical reasoning intersect cleanly in Venn diagrams. Below is a conceptual representation of the subset relationship corresponding to universal quantification ($A \subseteq B$).



5 Key Takeaways for Examinations

- Always verify the domain of discourse in quantified statements ($\forall$ vs. $\exists$).
- Remember that the contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$, and they are strictly equivalent.
- Avoid the fallacy of affirming the consequent: knowing $P \implies Q$ and Q does *not* imply P.

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