Arithmetic Word Problems Cheat Sheet

Master Framework, Formulas, and Problem-Solving Strategies

1. The 4-Step Problem Framework

Mastering word problems requires a systematic approach to translate natural language into mathematical equations.

1. **Read & Comprehend:** Identify the goal and unknown variables.
2. **Assign Variables:** Let $x, y, \ldots$ represent unknown quantities.
3. **Translate to Equations:** Convert statements into algebraic expressions.
4. **Solve & Verify:** Perform calculations and check against context.

2. Percentage & Ratio Formulas

Essential relations for scaling, mixtures, and proportions:

- **Percentage Change:**

$$\% \text{ Change} = \frac{\text{New} - \text{Old}}{\text{Old}} \times 100$$

- **Simple Interest:**

$$SI = \frac{P \times R \times T}{100}$$

- **Compound Interest:**

$$A = P\left(1 + \frac{R}{100}\right)^T$$

- **Direct Proportion:** If $y \propto x$, then $\frac{y_1}{x_1} = \frac{y_2}{x_2}$
- **Inverse Proportion:** If $y \propto \frac{1}{x}$, then $x_1 y_1 = x_2 y_2$

3. Speed, Distance, & Time

Fundamental kinematic relations and averages:

- **Core Formula:**

$$\text{Distance} = \text{Speed} \times \text{Time}$$

- **Average Speed (Equal Distances):**

$$v_{avg} = \frac{2v_1 v_2}{v_1 + v_2}$$

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- **Relative Speed:**
  - Same direction: $v_1 - v_2$
  - Opposite direction: $v_1 + v_2$
- **Trains (Length L_1, L_2, Speeds v_1, v_2):**

$$\text{Time} = \frac{L_1 + L_2}{v_1 \pm v_2}$$

4. Work & Time Problems

Core principle: Work equals Rate multiplied by Time.

- If A completes a job in n days, A's 1-day work is $\frac{1}{n}$.
- **Combined Work:** If A takes a days and B takes b days, working together they finish in:

$$T = \frac{a \cdot b}{a + b}$$

- **Pipes & Cisterns:** Inlet pipes do positive work (+), outlet/leak pipes do negative work (−).

5. Mixtures & Alligations

Used to find the ratio in which two ingredients at different prices must be mixed to get a mixture of a desired mean price.

Rule of Alligation:

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{C_d - M}{M - C_c}$$

where C_c is cost of cheaper, C_d is cost of dearer, and M is mean price.

6. Quick Tips & Shortcuts

- **Ages:** The difference between the ages of two people remains constant across all time periods.
- **Partnerships:** Profit is always distributed in the ratio of the product of capital invested and time duration (Investment × Time).
- **Unit Conversion:** Always ensure units match (e.g., convert km/hr to m/s by multiplying by $\frac{5}{18}$).