

1. Exponents and Logarithms

Exponent Laws

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$(ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n}, \quad a^{1/n} = \sqrt[n]{a}$$

Logarithm Properties

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^k) = k \log_b x$$

$$\text{Change of Base: } \log_b x = \frac{\log_c x}{\log_c b}$$

2. Polynomials and Factoring

Special Products

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

Quadratic Formula

For $ax^2 + bx + c = 0$ ($a \neq 0$):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant $\Delta = b^2 - 4ac$:

- $\Delta > 0$: Two distinct real roots
- $\Delta = 0$: One repeated real root
- $\Delta < 0$: Two complex conjugate roots

3. Sequences and Series

Arithmetic Progression (AP)

n -th term:

$$a_n = a_1 + (n - 1)d$$

Sum of first n terms:

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d] = \frac{n}{2}(a_1 + a_n)$$

Geometric Progression (GP)

n -th term:

$$a_n = a_1 r^{n-1}$$

Sum of first n terms:

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \quad (r \neq 1)$$

Sum of infinite GP ($|r| < 1$):

$$S_\infty = \frac{a_1}{1 - r}$$

4. Complex Numbers

Fundamentals

Defined as $z = x + iy$ where $i^2 = -1$.

- Magnitude: $|z| = \sqrt{x^2 + y^2}$
- Conjugate: $\bar{z} = x - iy$
- Polar Form: $z = r(\cos \theta + i \sin \theta) = re^{i\theta}$
- Euler's Formula: $e^{i\theta} = \cos \theta + i \sin \theta$

5. Binomial Theorem

Expansion

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

Where the binomial coefficient is:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

6. Matrices and Determinants