Quadratic Equations: Ultimate Formula Sheet

Master reference sheet for academic excellence

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1. Standard Form Basics

A quadratic equation in one variable is a polynomial equation of degree 2, generally written as:

$$\mathbf{ax^2 + bx + c = 0}$$

where $a, b, c \in \mathbb{R}$ and $a \neq 0$.
* **a**: Leading coefficient (determines parabola width/direction). * **b**: Linear coefficient. * **c**: Constant term (y-intercept).

2. The Quadratic Formula

The solutions (roots) for any quadratic equation are given by the universal formula:

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Derived via completing the square, this formula handles all real and complex roots effortlessly.

The expression inside the square root is the discriminant:

$$\mathbf{\Delta = b^2 - 4ac}$$

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* $\mathbf{\Delta > 0}$: Two distinct real roots. * $\mathbf{\Delta = 0}$: Exactly one real root (repeated/double root). * $\mathbf{\Delta < 0}$: Two complex conjugate roots.

4. Roots Coefficients

Let x_1 and x_2 be the roots of $ax^2 + bx + c = 0$:
* **Sum of Roots:**

$$x_1 + x_2 = -\frac{b}{a}$$

* **Product of Roots:**

$$x_1 x_2 = \frac{c}{a}$$

— * **Difference of Roots:**

$$|x_1 - x_2| = \frac{\sqrt{\Delta}}{|a|}$$

5. Vertex Form of a Parabola

A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in vertex form:

$$\mathbf{f(x) = a(x - h)^2 + k}$$

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* Vertex coordinates: $(h, k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ * Axis of symmetry: $x = h = -\frac{b}{2a}$

6. Important Identities

* $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2$ * $(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$ * $x_1^3 + x_2^3 = (x_1 + x_2)(x_1^2 - x_1x_2 + x_2^2)$