1. Vector Calculus Basics

Gradient (Cartesian):

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

Divergence:

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Curl:

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Product Rules Identities: * $\nabla(fg) = f\nabla g + g\nabla f$ * $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot \nabla f$ * $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times \nabla f$ * $\nabla \times (\nabla f) = 0$ * $\nabla \cdot (\nabla \times \mathbf{A}) = 0$
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2. Electrostatics

Coulomb's Law:

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

Electric Field:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{r^2}\hat{\mathbf{r}}\, d\tau'$$

Gauss's Law (Integral Form):

$$\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

Gauss's Law (Differential Form):

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

Electrostatic Potential:

$$V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$$

$$\mathbf{E} = -\nabla V$$

Poisson's Laplace's Equations:

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad \text{(Poisson)}$$

$$\nabla^2 V = 0 \quad \text{(Laplace)}$$

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3. Magnetostatics

Lorentz Force Law:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Biot-Savart Law:

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

Ampere's Law (Integral Form):

$$\oint_{\mathcal{P}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$$

Ampere's Law (Differential Form):

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Magnetic Vector Potential A:

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \cdot \mathbf{A} = 0 \quad \text{(Coulomb Gauge)}$$

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4. Electrodynamics Maxwell's Eqns

Faraday's Law:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampere-Maxwell Law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t}$$

Maxwell's Equations Summary: 1. $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ 2. $\nabla \cdot \mathbf{B} = 0$ 3. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ 4. $\nabla \times \mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t}$
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5. Waves Radiation

Wave Equation in Vacuum:

$$\nabla^2 \mathbf{E} = \mu_0\epsilon_0\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \mu_0\epsilon_0\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Speed of Light:

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Poynting Vector (Energy Flux):

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$$

Electromagnetic Momentum:

$$\mathbf{g} = \mu_0\epsilon_0\mathbf{S} = \frac{\mathbf{S}}{c^2}$$