

Lesson Summary: Fundamental Arithmetic

Comprehensive Guide Core Concepts

1 Introduction to Number Systems

Arithmetic is the oldest and most elementary branch of mathematics, dealing with the properties and manipulation of numbers. We begin our study with the standard classification of numbers:

- **Natural Numbers ($\mathbb{N}$):** Counting numbers, $\mathbb{N} = \{1, 2, 3, \dots\}$.
- **Integers ($\mathbb{Z}$):** Whole numbers including negatives, $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$.
- **Rational Numbers ($\mathbb{Q}$):** Numbers expressible as a fraction $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$.
- **Real Numbers ($\mathbb{R}$):** All rational and irrational numbers forming a continuous continuum.

2 Operations and Fundamental Properties

Let $a, b, c \in \mathbb{R}$. The basic operations of addition and multiplication satisfy several fundamental laws:

1. Commutative Laws:

$$a + b = b + a \quad \text{and} \quad a \cdot b = b \cdot a$$

2. Associative Laws:

$$(a + b) + c = a + (b + c) \quad \text{and} \quad (a \cdot b) \cdot c = a \cdot (b \cdot c)$$

3. Distributive Law:

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

3 Divisibility, LCM, and GCD

Definition 3.1 (Greatest Common Divisor). *The Greatest Common Divisor (GCD) of two integers a and b , not both zero, is the largest positive integer that divides both a and b , denoted as $\gcd(a, b)$.*

For any two positive integers a and b , their relationship with the Least Common Multiple (LCM) is given by the elegant product formula:

$$\text{lcm}(a, b) \cdot \gcd(a, b) = |a \cdot b|$$

4 Fractions and Rational Expressions

When dealing with arithmetic fractions, operations require common denominators for addition, whereas multiplication combines numerators and denominators directly:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}, \quad \text{where } b, d \neq 0$$

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}, \quad \text{where } b, d \neq 0$$

End of Lesson Summary. Visit <https://www.exambhai.com> for practice problems and video solutions.

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