ExamBhai.com — Premium Study Material

Master Formula Sheet: Differential & Integral Calculus

1. LIMITS & CONTINUITY

Standard Limits

margin=-10pt

- $\lim_{x\to 0} \frac{\sin x}{x} = 1, \quad \lim_{x\to 0} \frac{\tan x}{x} = 1$
- $\lim_{x\to 0} \frac{1-\cos x}{x} = 0, \quad \lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$
- $\lim_{x\to 0} \frac{e^x-1}{x} = 1, \quad \lim_{x\to 0} \frac{a^x-1}{x} = \ln a$
- $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$
- $\lim_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}$
- $\lim_{x\to \infty} (1+\frac{k}{x})^x = e^k$

L'Hôpital's Rule

If $\lim \frac{f(x)}{g(x)}$ yields $[\frac{0}{0}]$ or $[\frac{\infty}{\infty}]$:

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

2. DERIVATIVES & RULES

Fundamental Rules

margin=-10pt

- **Product Rule:** $(uv)' = u'v + uv'$
- **Quotient Rule:** $(\frac{u}{v})' = \frac{u'v-uv'}{v^2}$
- **Chain Rule:** $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

Common Derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a \qquad \frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Inverse Trig Derivatives

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\text{arcsec } x) = \frac{1}{|x|\sqrt{x^2-1}}$$

3. APPLICATIONS OF DERIVATIVES

Taylor & Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Maclaurin Series ($a = 0$):

margin=-10pt

- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

Mean Value Theorem (MVT)

If f is continuous on $[a, b]$ and differentiable on (a, b), $\exists c \in (a, b)$ such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

4. INDEFINITE INTEGRALS

Standard Integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$
$$\int \frac{1}{x}\,dx = \ln|x| + C$$
$$\int e^x\,dx = e^x + C, \int a^x\,dx = \frac{a^x}{\ln a} + C$$
$$\int \sin x\,dx = -\cos x + C, \quad \int \cos x\,dx = \sin x + C$$
$$\int \sec^2 x\,dx = \tan x + C, \quad \int \csc^2 x\,dx = -\cot x + C$$
$$\int \sec x \tan x\,dx = \sec x + C$$

Special Forms

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan(\frac{x}{a}) + C$$
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin(\frac{x}{a}) + C$$
$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\text{arcsec}(\frac{|x|}{a}) + C$$
$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln|\frac{a + x}{a - x}| + C$$

5. INTEGRATION TECHNIQUES

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Selection Priority (ILATE): Inverse Trig, Logarithmic, Algebraic, Trigonometric, Exponential.

Trigonometric Substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

6. DEFINITE INTEGRALS & FTC

Fundamental Theorem of Calculus

margin=-10pt

- **FTC 1:** $\frac{d}{dx}[\int_a^x f(t)\,dt] = f(x)$
- **FTC 2:** $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F'(x) = f(x)$

Leibniz Rule

$$\frac{d}{dx}[\int_{g(x)}^{h(x)} f(t)\,dt] = f(h(x))h'(x) - f(g(x))g'(x)$$

7. APPLICATIONS OF INTEGRATION

Area, Arc Length & Surface Area

- **Area between curves:** $A = \int_a^b |f(x) - g(x)|\, dx$
- **Arc Length:** $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$
- **Surface Area of Revolution:**

$$S = 2\pi \int_a^b y\sqrt{1 + (\frac{dy}{dx})^2}\, dx \quad \text{(about x-axis)}$$

Volumes of Solids of Revolution

margin=-10pt

- **Disk Method:** $V = \pi \int_a^b [f(x)]^2\, dx$
- **Washer Method:** $V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$
- **Shell Method:** $V = 2\pi \int_a^b x f(x)\, dx$

8. MULTIVARIABLE CALCULUS

Gradient, Divergence & Curl

margin=-10pt

- **Gradient:** $\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$
- **Divergence:** $\nabla \cdot \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$
- **Curl:** $\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$

Integral Theorems

margin=-10pt

- **Green's Theorem:**

$$\oint_C (P\, dx + Q\, dy) = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) dA$$

- **Divergence Theorem:**

$$\iint_{\partial V} \mathbf{F} \cdot d\mathbf{S} = \iiint_V (\nabla \cdot \mathbf{F})\, dV$$

- **Stokes' Theorem:**

$$\oint_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$