

Algebra Master Formula Sheet

Comprehensive Reference for Advanced Algebraic Structures and Identities

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1. Fundamental Expansions

$$\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2 \\ (a-b)^2 &= a^2 - 2ab + b^2 \\ a^2 - b^2 &= (a-b)(a+b) \\ (a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\ a^3 \pm b^3 &= (a \pm b)(a^2 \mp ab + b^2)\end{aligned}$$

2. Quadratic Equations

$ax^2 + bx + c = 0$, the roots are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Delta = b^2 - 4ac$: * $\Delta > 0$: Two distinct real roots. * $\Delta = 0$: Exactly one repeated real root. * $\Delta < 0$: Two complex conjugate roots.'s Formulas for roots r_1, r_2 :

$$r_1 + r_2 = -\frac{b}{a}, \quad r_1 r_2 = \frac{c}{a}$$

3. Exponents and Logarithms

$$\begin{aligned}a^m \cdot a^n &= a^{m+n} \\ \frac{a^m}{a^n} &= a^{m-n} \\ (a^m)^n &= a^{mn} \\ a^{-m} &= \frac{1}{a^m}\end{aligned}$$

$$\begin{aligned}\log_b(xy) &= \log_b x + \log_b y \\ \log_b\left(\frac{x}{y}\right) &= \log_b x - \log_b y \\ \log_b(x^k) &= k \log_b x \\ \log_b x &= \frac{\log_c x}{\log_c b}\end{aligned}$$

4. Binomial Theorem

the binomial coefficient is defined as:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

5. Progressions and Series

Arithmetic Progression (AP):* n -th term: $a_n = a_1 + (n-1)d$ * Sum of n terms: $S_n = \frac{n}{2}[2a_1 + (n-1)d]$ **Geometric Progression (GP):*** n -th term: $a_n = a_1 r^{n-1}$ * Sum of n terms: $S_n = \frac{a_1(1-r^n)}{1-r}$ ($r \neq 1$)* Infinite sum ($|r| < 1$): $S_\infty = \frac{a_1}{1-r}$

6. Matrices and Determinants

a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$\det(A) = ad - bc$$

matrix A^{-1} :

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

that $ad - bc \neq 0$.