Algebra Master Formula Sheet

Comprehensive Reference for Advanced Algebraic Structures and Identities

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1. Exponents and Logarithms

$$a^m \cdot a^n = a^{m+n} \qquad \log_b(xy) = \log_b x + \log_b y$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$(a^m)^n = a^{mn} \qquad \log_b(x^k) = k \log_b x$$

$$a^{-m} = \frac{1}{a^m} \qquad \log_b x = \frac{\log_a x}{\log_a b}$$

$$a^{1/n} = \sqrt[n]{a} \qquad \log_b 1 = 0,\ \log_b b = 1$$

2. Quadratic Equations

For a quadratic equation $ax^2 + bx + c = 0$:

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant (Δ):

$$\Delta = b^2 - 4ac$$

* $\Delta > 0$: Two distinct real roots * $\Delta = 0$: One repeated real root * $\Delta < 0$: Two complex conjugate roots

Vieta's Formulas:

$$\text{Sum} = x_1 + x_2 = -\frac{b}{a}, \quad \text{Product} = x_1 x_2 = \frac{c}{a}$$

3. Polynomial Expansions

$$
\begin{aligned}
(a+b)^2 &= a^2 + 2ab + b^2 \\
(a-b)^2 &= a^2 - 2ab + b^2 \\
(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
(a-b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 \\
a^2 - b^2 &= (a-b)(a+b) \\
a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\
a^3 - b^3 &= (a-b)(a^2 + ab + b^2)
\end{aligned}
$$

4. Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Where the binomial coefficient is defined as:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The general term (r-th term from the beginning) in the expansion of $(x+y)^n$ is given by:

$$T_{r+1} = \binom{n}{r} x^{n-r} y^r$$

5. Sequences and Series

Arithmetic Progression (AP):

$$
\begin{aligned}
a_n &= a_1 + (n-1)d \\
S_n &= \frac{n}{2}[2a_1 + (n-1)d] = \frac{n}{2}(a_1 + a_n)
\end{aligned}
$$

Geometric Progression (GP):

$$
\begin{aligned}
a_n &= a_1 r^{n-1} \\
S_n &= \frac{a_1(1-r^n)}{1-r} \quad (r \neq 1) \\
S_\infty &= \frac{a_1}{1-r} \quad (|r| < 1)
\end{aligned}
$$

6. Complex Numbers

Let $z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$.

* **Modulus:** $|z| = \sqrt{x^2 + y^2}$ * **Argument:** $\theta = \arctan\left(\frac{y}{x}\right)$ * **Conjugate:** $\bar{z} = x - iy$ * **Euler's Formula:** $e^{i\theta} = \cos\theta + i\sin\theta$ * **De Moivre's Theorem:**

$$(r(\cos\theta + i\sin\theta))^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

7. Matrices and Determinants

For a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

* **Determinant:**

$$\det(A) = ad - bc$$

* **Inverse:**

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$