1. Basics and Components

Let $\vec{a} = \langle a_1, a_2, a_3 \rangle = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$.

- **Magnitude (Norm):**
$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$
- **Unit Vector:** $\hat{a} = \frac{\vec{a}}{\|\vec{a}\|}$ (if $\|\vec{a}\| \neq 0$)
- **Position Vector:** Given $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$:
$$\vec{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$
- **Direction Cosines:** $\cos\alpha = \frac{a_1}{\|\vec{a}\|}$, $\cos\beta = \frac{a_2}{\|\vec{a}\|}$, $\cos\gamma = \frac{a_3}{\|\vec{a}\|}$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

2. Dot Product (Scalar Product)

$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\|\cos\theta$

- **Component Form:**
$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$
- **Orthogonality:** $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$
- **Angle Between Vectors:**
$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\|\vec{b}\|}$$
- **Scalar Projection of $\vec{a}$ on $\vec{b}$:**
$$\text{comp}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$$
- **Vector Projection of $\vec{a}$ on $\vec{b}$:**
$$\text{proj}_{\vec{b}}\vec{a} = \left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|^2}\right)\vec{b}$$

3. Cross Product (Vector Product)

$\vec{a} \times \vec{b} = (\|\vec{a}\|\|\vec{b}\|\sin\theta)\hat{n}$

- **Determinant Form:**
$$\vec{a} \times \vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
- **Parallelism:** $\vec{a} \parallel \vec{b} \iff \vec{a} \times \vec{b} = \vec{0}$
- **Geometric Meaning:** $\|\vec{a} \times \vec{b}\|$ equals the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$.
- **Area of Triangle:** $\frac{1}{2}\|\vec{a} \times \vec{b}\|$

4. Triple Products

- **Scalar Triple Product (Box Product):**
$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$
Geometric Meaning: Volume of parallelpiped formed by $\vec{a}, \vec{b}, \vec{c}$. Coplanar if equal to 0.
- **Vector Triple Product:**
$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

5. Vector Calculus (Brief)

- **Vector Derivative:** $\frac{d}{dt}\vec{r}(t) = \langle x'(t), y'(t), z'(t) \rangle$
- **Gradient:** $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$
- **Divergence:** $\nabla \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$
- **Curl:** $\nabla \times \vec{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$