1. BASICS & LINE SEGMENTS

Distance & Midpoint

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points.

- **Distance Formula:**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- **Midpoint Formula:**

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Section Formula

Dividing segment PQ in ratio $m : n$:

- **Internal Division:**

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

- **External Division:**

$$P(x, y) = \left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}\right)$$

Area of Polygon

Triangle Area with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$:

$$\text{Area} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Points are **collinear** if Area $= 0$.

2. STRAIGHT LINES

Forms of Line Equations

- **Slope-Intercept:** $y = mx + c$
- **Point-Slope:** $y - y_1 = m(x - x_1)$
- **Two-Point:** $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- **Intercept Form:** $\frac{x}{a} + \frac{y}{b} = 1$
- **Normal Form:** $x \cos\alpha + y \sin\alpha = p$
- **General Form:** $Ax + By + C = 0$

Angles & Distances

For line $Ax + By + C = 0$:

- **Slope:** $m = -\frac{A}{B}$
- **Angle between two lines:** $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$
- **Parallel Lines:** $m_1 = m_2$
- **Perpendicular Lines:** $m_1 \cdot m_2 = -1$
- **Distance from (x_0, y_0) to Line:**

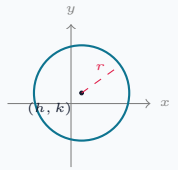
$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

- **Distance between Parallel Lines:**

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

3. CIRCLES

Equations of a Circle



- **Standard Form:** $(x - h)^2 + (y - k)^2 = r^2$
- **General Form:** $x^2 + y^2 + 2gx + 2fy + c = 0$
  - Center $= (-g, -f)$
  - Radius $r = \sqrt{g^2 + f^2 - c}$

Tangents & Normals

Tangent to $x^2 + y^2 = r^2$ at point (x_1, y_1):

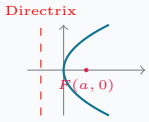
$$xx_1 + yy_1 = r^2$$

Condition for Tangency: Line $y = mx + c$ is tangent if $c = \pm r\sqrt{1 + m^2}$.

4. CONIC SECTIONS

Parabola

Standard Equation: $y^2 = 4ax$ $(a > 0)$



- **Focus:** $(a, 0)$, **Directrix:** $x = -a$
- **Latus Rectum Length:** $4a$
- **Eccentricity:** $e = 1$
- **Tangent at (x_1, y_1):** $yy_1 = 2a(x + x_1)$

Ellipse

Standard Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$

- **Foci:** $(\pm ae, 0)$
- **Eccentricity:** $e = \sqrt{1 - \frac{b^2}{a^2}}$
- **Directrices:** $x = \pm \frac{a}{e}$
- **Latus Rectum:** $\frac{2b^2}{a}$
- **Focal Distance Sum:** $PF_1 + PF_2 = 2a$

Hyperbola

Standard Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

- **Foci:** $(\pm ae, 0)$
- **Eccentricity:** $e = \sqrt{1 + \frac{b^2}{a^2}}$
- **Asymptotes:** $y = \pm \frac{b}{a} x$
- **Rectangular Hyperbola:** $xy = c^2$ $(e = \sqrt{2})$

5. CENTERS OF A TRIANGLE

Triangle Centers Setup

For vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ with opposite side lengths a, b, c:

- **Centroid (G):** Intersection of medians.

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

- **Incenter (I):** Intersection of angle bisectors.

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$$

- **Orthocenter (H):** Intersection of altitudes.
- **Circumcenter (O):** Intersection of $\perp$ bisectors.
- **Euler Line:** O, G, H are collinear, and G divides HO in ratio $2 : 1$.