Coordinate Geometry — Comprehensive Formula Sheet

Master reference guide for 2D analytic geometry

1. Point Line Fundamentals

Distance Formula: For points $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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Section Formula: Internal division for ratio $m : n$:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

External division $(m : n)$:

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Midpoint Formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Centroid of a Triangle:

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Area of a Triangle:

$$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

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2. Straight Lines

Slope (Gradient) (m):

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

Forms of Equations of a Line: * **Slope-Intercept**: $y = mx + c$ * **Point-Slope**: $y - y_1 = m(x - x_1)$ * **Two-Point**: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ * **Intercept Form**: $\frac{x}{a} + \frac{y}{b} = 1$ * **Normal Form**: $x\cos\alpha + y\sin\alpha = p$ * **General Form**: $Ax + By + C = 0$

Angle Between Two Lines:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

* Parallel: $m_1 = m_2$ * Perpendicular: $m_1 m_2 = -1$

Perpendicular Distance: From point (x_1, y_1) to $Ax + By + C = 0$:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

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3. Pair of Straight Lines

Homogeneous Equation of 2nd Degree:

$$ax^2 + 2hxy + by^2 = 0$$

Represents two lines passing through the origin. * Angle between lines: $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$ * Lines coincide if $h^2 = ab$ * Lines perpendicular if $a + b = 0$

General Second Degree Equation:

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Represents a pair of straight lines if:

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

and $h^2 \geq ab$.

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4. Circles

Standard Equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

Center (h, k), radius r.

General Equation:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

* Center: $(-g, -f)$ * Radius: $r = \sqrt{g^2 + f^2 - c}$

Diameter Form:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Tangent Equations: * Point (x_1, y_1) on circle $x^2 + y^2 = r^2$:

$$xx_1 + yy_1 = r^2$$

* Slope form $(y = mx + c)$:

$$c = \pm r\sqrt{1 + m^2}$$

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5. Conic Sections — General

Defined by eccentricity e: Ratio of distance from focus to distance from directrix. * $e = 1$: Parabola * $e < 1$: Ellipse * $e > 1$: Hyperbola

General Conic Equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Discriminant $\Delta = B^2 - 4AC$: * $\Delta < 0$: Ellipse (or circle/point/none) * $\Delta = 0$: Parabola (or parallel lines/none) * $\Delta > 0$: Hyperbola (or intersecting lines)

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6. Parabola

Standard equations and properties:

Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$
Directrix	$x = -a$	$x = a$	$y = -a$
Axis	$y = 0$	$y = 0$	$x = 0$
Tangent at t	$yt = x + at^2$	–	$xt = y + at^2$

Parametric Coordinates: $(at^2, 2at)$ for $y^2 = 4ax$.

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7. Ellipse

Standard Equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$$

* **Vertices**: $(\pm a, 0)$ * **Foci**: $(\pm ae, 0)$ * **Eccentricity** (e): $e = \sqrt{1 - \frac{b^2}{a^2}}$ * **Directrices**: $x = \pm \frac{a}{e}$ * **Major Axis Length**: $2a$ * **Minor Axis Length**: $2b$ * **Latus Rectum**: $\frac{2b^2}{a}$ * **Parametric Form**: $(a\cos\theta, b\sin\theta)$

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8. Hyperbola

Standard Equation:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

* **Vertices**: $(\pm a, 0)$ * **Foci**: $(\pm ae, 0)$ * **Eccentricity** (e): $e = \sqrt{1 + \frac{b^2}{a^2}}$ * **Directrices**: $x = \pm \frac{a}{e}$ * **Transverse Axis**: $2a$, **Conjugate Axis**: $2b$ * **Latus Rectum**: $\frac{2b^2}{a}$ * **Asymptotes**: $\frac{x}{a} \pm \frac{y}{b} = 0$ * **Parametric Form**: $(a\sec\theta, b\tan\theta)$

Rectangular Hyperbola: When $a = b$, $x^2 - y^2 = a^2$ (or $xy = c^2$). Eccentricity is $\sqrt{2}$.