

Ultimate Calculus Cheat Sheet

Comprehensive formulas for Limits, Derivatives, Integrals, and Series

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1. Limits Continuity

Definition of Derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

L'Hôpital's Rule: If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$, then:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

2. Derivative Rules

Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$

Product Rule: $(fg)' = f'g + fg'$

Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

3. Derivatives of Trig Functions

- $\frac{d}{dx}[\sin x] = \cos x$
- $\frac{d}{dx}[\cos x] = -\sin x$
- $\frac{d}{dx}[\tan x] = \sec^2 x$
- $\frac{d}{dx}[\csc x] = -\csc x \cot x$
- $\frac{d}{dx}[\sec x] = \sec x \tan x$
- $\frac{d}{dx}[\cot x] = -\csc^2 x$

4. Integration Techniques

Integration by Parts:

$$\int u \, dv = uv - \int v \, du$$

Substitution:

$$\int f(g(x))g'(x) \, dx = \int f(u) \, du$$

5. Common Integrals

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int e^x dx = e^x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sec^2 x dx = \tan x + C$

6. Series Taylor Expansions

Geometric Series: $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \quad (|r| < 1)$

Taylor Series:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

Common Maclaurin Series:

- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots$
- $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
- $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$