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Modern Mathematics Cheat Sheet

1. Set Theory & Relations

Definitions:

- **Power Set:** $\mathcal{P}(A) = \{B : B \subseteq A\}$, $|\mathcal{P}(A)| = 2^{|A|}$.
- **Cartesian Product:** $A \times B = \{(a, b) : a \in A, b \in B\}$.

Properties of Relations:

- **Reflexive:** $(a, a) \in R \forall a \in A$.
- **Symmetric:** $(a, b) \in R \implies (b, a) \in R$.
- **Transitive:** $(a, b) \in R \wedge (b, c) \in R \implies (a, c) \in R$.
- **Equivalence:** Reflexive + Symmetric + Transitive.

- **Hamiltonian Cycle:** Visits every *vertex* exactly once.

Handshaking Lemma:

$$\sum_{v \in V} \deg(v) = 2|E|$$

4. Mathematical Logic

Equivalences:

- **Implication:** $P \implies Q \equiv \neg P \vee Q$.
- **Contrapositive:** $P \implies Q \equiv \neg Q \implies \neg P$.
- **De Morgan's Laws:** $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$, $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$.

2. Abstract Algebra

Groups $(G, \cdot)$:

1. Closure: $\forall a, b \in G, a \cdot b \in G$.
2. Associativity: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
3. Identity: $\exists e \in G$ s.t. $a \cdot e = e \cdot a = a$.
4. Inverse: $\forall a \in G, \exists a^{-1} \in G$ s.t. $a \cdot a^{-1} = e$.

Subgroups & Lagrange's Theorem: H is a subgroup of a finite group G , then $|H|$ divides $|G|$.

3. Graph Theory

Eulerian vs. Hamiltonian:

- **Eulerian Path/Circuit:** Visits every *edge* exactly once. Exists iff connected and vertices have even degrees (for circuit).