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Modern Mathematics Cheat Sheet

1. Set Theory & Mathematical Logic

De Morgan's Laws:

$$(A \cup B)^c = A^c \cap B^c, \quad (A \cap B)^c = A^c \cup B^c$$

Quantifiers & Logic Identities:

- Implication: $P \implies Q \equiv \neg P \vee Q$
- Contrapositive: $P \implies Q \equiv \neg Q \implies \neg P$

Zermelo-Fraenkel Axiom (Axiom of Choice):

$$\prod_{i \in I} X_i \neq \emptyset \quad \text{for non-empty sets } X_i$$

2. Abstract Algebra Group Theory:

- Axioms: Closure, Associativity, Identity (e), Inverse (g^{-1}).
- **Lagrange's Theorem:** For finite group G and subgroup $H \leq G$, $|H|$ divides $|G|$.
- **First Isomorphism Theorem:** $G/\ker(\varphi) \cong \text{Im}(\varphi)$

Ring & Field Theory:

- Ideal $I \triangleleft R$: $rI \subseteq I, Ir \subseteq I \quad \forall r \in R$.
- Field $\mathbb{F}$: Commutative ring where every non-zero element has a multiplicative inverse.

3. Topology & Metric Spaces

Space Definition: $d : X \times X \rightarrow \mathbb{R}_{\geq 0}$ satisfies:

1. $d(x, y) = 0 \iff x = y$
2. $d(x, y) = d(y, x)$
3. $d(x, z) \leq d(x, y) + d(y, z)$ (Triangle Inequality)

Topological Space (X, τ) :

- $\emptyset, X \in \tau$
- Arbitrary unions belong to τ .
- Finite intersections belong to τ .

Compactness: Every open cover of K has a finite subcover. In $\mathbb{R}^n$, K is compact $\iff$ closed & bounded (Heine-Borel).

4. Linear Algebra & Vector Spaces

Eigenvalues & Eigenvectors:

$$A\mathbf{v} = \lambda\mathbf{v} \implies \det(A - \lambda I) = 0$$

Spectral Theorem: If A is a real symmetric matrix, then $A = V\Lambda V^T$ where V is orthogonal and Λ is diagonal.

Singular Value Decomposition (SVD):

$$A = U\Sigma V^*$$

where U, V are unitary and Σ contains singular values $\sigma_i = \sqrt{\lambda_i(A^*A)}$.

5. Complex Analysis

Cauchy-Riemann Equations: For $f(z) = u(x, y) + iv(x, y)$ to be analytic:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Cauchy's Integral Formula:

$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - a} dz$$

Residue Theorem:

$$\oint_C f(z) dz = 2\pi i \sum \text{Res}(f, z_k)$$

6. Differential Geometry & Tensor Calculus

Metric Tensor & Line Element:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Christoffel Symbols of Second Kind:

$$\Gamma_{\mu\nu}^\sigma = \frac{1}{2} g^{\sigma\lambda} \left(\frac{\partial g_{\nu\lambda}}{\partial x^\mu} + \frac{\partial g_{\mu\lambda}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\lambda} \right)$$

Riemann Curvature Tensor:

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma_{\nu\sigma}^\rho - \partial_\nu \Gamma_{\mu\sigma}^\rho + \Gamma_{\mu\lambda}^\rho \Gamma_{\nu\sigma}^\lambda - \Gamma_{\nu\lambda}^\rho \Gamma_{\mu\sigma}^\lambda$$

7. Measure Theory & Probability

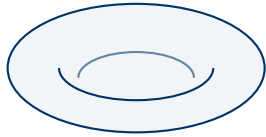
Definition of σ -algebra $\mathcal{F}$:

- $X \in \mathcal{F}$
- $A \in \mathcal{F} \implies A^c \in \mathcal{F}$
- Countable union closure: $A_i \in \mathcal{F} \implies \bigcup_{i=1}^\infty A_i \in \mathcal{F}$

Lebesgue Dominated Convergence Theorem: If $f_n \rightarrow f$ a.e. and $|f_n| \leq g$ with $g \in L^1$, then:

$$\lim_{n \rightarrow \infty} \int X f_n d\mu = \int X f d\mu$$

8. Geometric Visualization



Torus Topology $\mathbb{T}^2 = S^1 \times S^1$

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