

1. Fundamental Translation Average Speed Guide

Translating natural language into mathematical expressions is the cornerstone of solving arithmetic word problems.

English Phrase	Math Operation
Sum, increased by, more than	Addition (+)
Difference, decreased by, less	Subtraction (-)
Product, of, times	Multiplication (×)
Quotient, per, ratio	Division (/)
Is, equals, results in	Equality (=)

Example: "Five more than twice a number is 17."
 $\rightarrow 2x + 5 = 17$.

2. Percentage Ratio Mastery

Percentages represent parts per hundred. Let P be the part, W be the whole, and R be the rate.

$$\text{Part} = \text{Whole} \times \frac{\text{Rate}}{100}$$

Percentage Increase/Decrease

$$\text{Percentage Change} = \frac{\text{New} - \text{Old}}{\text{Old}} \times 100\%$$

Proportions

If $\frac{a}{b} = \frac{c}{d}$, then cross-multiplication yields:

$$a \cdot d = b \cdot c$$

3. Speed, Distance, and Time

The relationship between speed (s), distance (d), and time (t) is governed by:

$$d = s \times t \quad \Rightarrow \quad s = \frac{d}{t}, \quad t = \frac{d}{s}$$

For two equal distances traveled at speeds v_1 and v_2 :

$$\text{Average Speed} = \frac{2v_1v_2}{v_1 + v_2}$$

Relative Speed

*** Same Direction:** $s_{\text{rel}} = |v_1 - v_2|$ *** Opposite Direction:** $s_{\text{rel}} = v_1 + v_2$

4. Work and Time Problems

If person A can complete a job in a hours, their 1-hour work rate is $\frac{1}{a}$.

$$\text{Combined Rate} = \frac{1}{a} + \frac{1}{b} = \frac{1}{t_{\text{total}}}$$

Tip: Always convert rates to work completed per unit time before combining.

5. Mixture and Alligation

To find the concentration of a mixture formed by mixing two quantities Q_1 and Q_2 with concentrations C_1 and C_2 :

$$\text{Final Concentration} = \frac{Q_1C_1 + Q_2C_2}{Q_1 + Q_2}$$

Alligation Rule

Ratio of quantities of cheaper to dearer ingredients:

$$\frac{Q_1}{Q_2} = \frac{C_2 - \text{Mean}}{\text{Mean} - C_1}$$

6. Simple Compound Interest

*** Simple Interest (SI):**

$$SI = \frac{P \times R \times T}{100}$$

where P = Principal, R = Rate/annum, T = Time.

*** Compound Interest (CI):**

$$A = P \left(1 + \frac{R}{100} \right)^t$$

$$CI = A - P$$