

Mastering Number Series

Elite Strategies, Patterns, and Shortcuts for Competitive Exams

1. Arithmetic Geometric Progressions

Arithmetic Progression (AP): Constant difference (d) between consecutive terms.

$$a_n = a_1 + (n - 1)d$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

* **Key check:** Look at $T_2 - T_1, T_3 - T_2$.

Geometric Progression (GP): Constant ratio (r) between consecutive terms.

$$a_n = a_1 r^{n-1}$$

* **Key check:** Look at $T_2/T_1, T_3/T_2$.

2. Difference Double Difference

When numbers grow or shrink steadily and slowly, examine the differences.

* **Level 1 Difference:** $b_n = a_{n+1} - a_n$ * **Level 2 Difference:** If Level 1 is erratic, compute differences of differences.

Example: 2, 5, 10, 17, 26, (...)

Series: 2 5 10 17

Diff 1: 3 5 7

Diff 2: 2 2

Next diff = $7 + 2 = 9 \Rightarrow 26 + 9 = 35$.

3. Power Exponent Series

Always memorize squares up to 30 and cubes up to 20.

* **Pure Squares/Cubes:** n^2, n^3 * **Modified Powers:**
* $n^2 \pm 1$ or $n^2 \pm n$ * $n^3 \pm 1$ or $n^3 \pm n$

Example ($n^3 + n$): 2, 10, 30, 68, 130, (...)
* $1^3 + 1 = 2$ * $2^3 + 2 = 10$ * $3^3 + 3 = 30$ * $4^3 + 4 = 68$ * $5^3 + 5 = 130$ *
Next: $6^3 + 6 = 216 + 6 = 222$.

4. Product Mixed Series

When numbers grow rapidly, multiplication is involved.

* **Standard Form:** $a_{n+1} = a_n \times m \pm c$

Example ($imes2 + 1$): 3, 7, 15, 31, 63, (...)
* $3 \times 2 + 1 = 7$ * $7 \times 2 + 1 = 15$ * **Next:** $63 \times 2 + 1 = 127$.

5. Prime Fibonacci Variations

* **Prime Series:** Look out for non-composite numbers: 2, 3, 5, 7, 11, 13, 17, ... * **Fibonacci/Staircase:** $a_n = a_{n-1} + a_{n-2}$.

Example (Fibonacci): 1, 1, 2, 3, 5, 8, 13, (...)

6. Alternating Twin Series

Sometimes two independent series are interleaved into one.

Example: 10, 20, 12, 24, 14, 28, (...)
* **Series 1 (Odd pos):** 10, 12, 14, ... (+2)
* **Series 2 (Even pos):** 20, 24, 28, ... (+4)
* **Next term:** Position 7 belongs to Series 1 $\Rightarrow 14 + 2 = 16$.

7. Elite Exam Strategy

- Growth Rate Check:** * Slow $\rightarrow$ Differences. * Rapid $\rightarrow$ Multiplication/Powers. * Alternating $\rightarrow$ Two-tier series.
- Decimal/Fraction Series:** Look for numerators and denominators changing via separate rules.