

Arithmetic Word Problems Masterclass

Comprehensive Strategies, Formulas, and Shortcuts for Competitive Exams

1. Fundamental Translation Profit, Loss, and Discount Guide

English sentences into mathematical equations is the core of solving word problems.

- **Sum, Increased by, More than:** Addition (+)
- **Difference, Decreased by, Less than:** Subtraction (−)
- **Product, Of, Twice, Times:** Multiplication (×)
- **Quotient, Per, Ratio, Out of:** Division (÷)
- **Is, Are, Equals, Results in:** Equal sign (=)

$$\text{Profit (P)} = \text{SP} - \text{CP} \quad (\text{if } \text{SP} > \text{CP})$$

$$\text{Loss (L)} = \text{CP} - \text{SP} \quad (\text{if } \text{CP} > \text{SP})$$

$$\text{Profit \%} = \left(\frac{\text{Profit}}{\text{CP}} \right) \times 100$$

$$\text{Discount} = \text{Marked Price (MP)} - \text{SP}$$

3. Ratio, Proportion & Partnership

Direct & Inverse Proportion

- **Direct:** $y = kx \implies \frac{y_1}{x_1} = \frac{y_2}{x_2}$
- **Inverse:** $y = \frac{k}{x} \implies x_1 y_1 = x_2 y_2$

Partnership Rule

is shared in the ratio of the product of capital invested and time period:

$$\text{Ratio of Profit} = (C_1 \times T_1) : (C_2 \times T_2) : (C_3 \times T_3)$$

2. Percentage & Profit-Loss

Percentage Change

$$\text{Percentage Change} = \left(\frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \right) \times 100$$

Successive Percentages

a value changes by $a\%$ and then by $b\%$, the net percentage change is:

$$\text{Net Change} = \left(a + b + \frac{ab}{100} \right) \%$$

4. Averages, Mixtures & Alligation

Average Concepts

$$\text{Average} = \frac{\text{Sum of Observations}}{\text{Number of Observations}}$$

Weighted Average:

$$\text{Avg}_{\text{mix}} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Rule of Alligation

mix two ingredients of cost C_1 and C_2 (where $C_1 < \text{Mean} < C_2$) to get a mean cost M :

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{C_2 - M}{M - C_1}$$

$$\begin{array}{ccc} C_1 \text{ (Cheaper)} & & C_2 - M \\ & \searrow \nearrow & \\ & M \text{ (Mean)} & \\ & \nearrow \searrow & \\ C_2 \text{ (Dearer)} & & M - C_1 \end{array}$$

5. Time, Speed and Distance

Core Formulas

$$\text{Distance} = \text{Speed} \times \text{Time}, \quad \text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Conversion: } 1 \text{ m/s} = \frac{18}{5} \text{ km/hr and } 1 \text{ km/hr} = \frac{5}{18} \text{ m/s.}$$

Average Speed

equal distances are covered at speeds s_1 and s_2 :

$$\text{Average Speed} = \frac{2s_1 s_2}{s_1 + s_2}$$

Relative Speed

- **Same Direction:** $S_{rel} = |s_1 - s_2|$
 - **Opposite Direction:** $S_{rel} = s_1 + s_2$
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6. Time and Work / Pipes & Cisterns

Core Concepts

A can do a piece of work in n days, A's 1 day's work = $\frac{1}{n}$.

Efficiency & Time Relation

$$\text{Efficiency} \propto \frac{1}{\text{Time Taken}}$$

A is twice as efficient as B, A takes half the time of B to complete the same work.

Pipe Inlets and Outlets

- Inlet pipe fills tank (+)
- Outlet/Leak empties tank (-)
- Net work in 1 hour = $\sum(\text{Inlets}) - \sum(\text{Outlets})$