

Number Systems: Master Cheat Sheet

Comprehensive Reference Guide for Advanced Mathematics

1. Classification of Numbers

Number systems form the foundation of mathematics, expanding hierarchically:

- **Natural Numbers ($\mathbb{N}$):** $\{1, 2, 3, \dots\}$
- **Whole Numbers ($\mathbb{W}$):** $\{0, 1, 2, 3, \dots\}$
- **Integers ($\mathbb{Z}$):** $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- **Rational Numbers ($\mathbb{Q}$):** $\left\{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\right\}$ (Terminating or repeating decimals).
- **Irrational Numbers ($\mathbb{Q}'$):** Non-terminating, non-repeating decimals (e.g., $\pi, e, \sqrt{2}$).
- **Real Numbers ($\mathbb{R}$):** $\mathbb{Q} \cup \mathbb{Q}'$.
- **Complex Numbers ($\mathbb{C}$):** $\{a + bi \mid a, b \in \mathbb{R}, i^2 = -1\}$.

- **HCF (Highest Common Factor):** Greatest divisor of two or more integers.
- **LCM (Lowest Common Multiple):** Smallest positive integer divisible by all given numbers.
- For fractions $\frac{a}{b}$ and $\frac{c}{d}$:

$$\text{HCF} = \frac{\text{HCF}(a, c)}{\text{LCM}(b, d)}, \quad \text{LCM} = \frac{\text{LCM}(a, c)}{\text{HCF}(b, d)}$$

4. Base Number Systems

Conversion between bases uses place value powers:

- **Decimal (Base 10):** Digits 0 – 9.
- **Binary (Base 2):** Digits 0, 1. Example:

$$1011_2 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 11_{10}$$
- **Hexadecimal (Base 16):** Digits 0 – 9, A – F (where A = 10, ..., F = 15).

2. Divisibility Rules

Divisible by	Condition
2	Last digit is even $\in \{0, 2, 4, 6, 8\}$.
3	Sum of digits is divisible by 3.
4	Number formed by last 2 digits is divisible by 4.
5	Last digit is 0 or 5.
6	Divisible by both 2 and 3.
8	Last 3 digits are divisible by 8.
9	Sum of digits is divisible by 9.
11	Difference of alternate digit sums is divisible by 11.

5. Fundamental Theorem of Arithmetic

Every integer greater than 1 either is a prime number itself or can be represented as a unique product of prime numbers (up to the order of factors):

$$n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$$

where p_i are distinct primes and a_i are positive integers.

3. LCM and HCF (GCD)

For any two positive integers a and b :

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

6. Modular Arithmetic

Let n be a positive integer (modulus). Two integers a and b are congruent modulo n , denoted: