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ALGEBRA ULTRA-COMPACT CHEAT SHEET

1. Exponents & Radicals

Exponent Rules ($a, b > 0$):

[leftmargin=*]

- $a^m \cdot a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $(\frac{a}{b})^n = \frac{a^n}{b^n}$
- $a^{-n} = \frac{1}{a^n}, \quad a^0 = 1 \ (a \neq 0)$

Radical Rules:

[leftmargin=*]

- $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}, \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

2. Fundamental Identities

Polynomial Expansions:

[leftmargin=*]

- $(a \pm b)^2 = a^2 \pm 2ab + b^2$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

Factoring Formulas:

[leftmargin=*]

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- $a^3 + b^3 = (a+b)(a^2-ab+b^2)$
- $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \cdots + b^{n-1})$

3. Quadratic Equations

For $ax^2 + bx + c = 0 \quad (a \neq 0)$: **Quadratic Formula:**

$$x = \frac{-b \pm \sqrt{D}}{2a}, \quad \text{where } D = b^2 - 4ac$$

Discriminant (D) Analysis:

[leftmargin=*]

- $D > 0$: 2 real & distinct roots.
- $D = 0$: 2 real & equal roots ($x = -b/2a$).
- $D < 0$: 2 complex conjugate roots.

Vieta's Formulas: If r_1, r_2 are roots:

$$r_1 + r_2 = -\frac{b}{a}, \quad r_1 r_2 = \frac{c}{a}$$

Vertex Form: $y = a(x-h)^2 + k$

$$h = -\frac{b}{2a}, \quad k = c - \frac{b^2}{4a} = -\frac{D}{4a}$$

4. Logarithms

Definition: $\log_b(x) = y \iff b^y = x \quad (b > 0, b \neq 1, x > 0)$ **Logarithm Laws:**

[leftmargin=*]

- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b(x/y) = \log_b(x) - \log_b(y)$
- $\log_b(x^k) = k\log_b(x)$
- $\log_b(1) = 0, \quad \log_b(b) = 1$
- **Change of Base:** $\log_b(x) = \frac{\log_c(x)}{\log_c(b)} = \frac{\ln x}{\ln b}$
- $b^{\log_b(x)} = x, \quad \log_b(b^x) = x$

5. Sequences & Series

Arithmetic Progression (AP):

[leftmargin=*]

- n-th term: $a_n = a + (n-1)d$
- Sum of n terms: $S_n = \frac{n}{2}(2a+(n-1)d) = \frac{n}{2}(a+l)$

Geometric Progression (GP):

[leftmargin=*]

- n-th term: $a_n = ar^{n-1}$
- Sum of n terms: $S_n = a\frac{1-r^n}{1-r} \quad (r \neq 1)$
- Infinite GP sum ($|r| < 1$): $S_\infty = \frac{a}{1-r}$

Means Relation: AM $\geq$ GM $\geq$ HM

$$\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b} \quad (a, b > 0)$$

6. Complex Numbers

Form: $z = a + bi = r(\cos\theta + i\sin\theta) = re^{i\theta}$

[leftmargin=*]

- $i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$
- Modulus: $|z| = r = \sqrt{a^2 + b^2}$
- Conjugate: $\bar{z} = a - bi, \quad z\bar{z} = |z|^2$
- Argument: $\theta = \arg(z) = \arctan(b/a)$
- **De Moivre's Theorem:**

$$(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$$

- **Euler's Formula:** $e^{i\theta} = \cos\theta + i\sin\theta$

7. Binomial Theorem

For any positive integer n:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Where Binomial Coefficient:

$$\binom{n}{k} = C(n,k) = \frac{n!}{k!(n-k)!}$$

Properties:

[leftmargin=*]

- Total terms $= n + 1$
- $\sum_{k=0}^{n} \binom{n}{k} = 2^n$
- $\binom{n}{k} = \binom{n}{n-k}$
- General Term: $T_{k+1} = \binom{n}{k} x^{n-k} y^k$

8. Matrices & Determinants

For a 2×2 Matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

[leftmargin=*]

- Determinant: $\det(A) = |A| = ad - bc$
- Inverse: $A^{-1} = \frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad (|A| \neq 0)$

Cramer's Rule (2×2 **System):** $ax + by = e, \quad cx + dy = f$

$$x = \frac{ed - bf}{ad - bc}, \quad y = \frac{af - ec}{ad - bc}$$

Properties:

[leftmargin=*]

- $(AB)^{-1} = B^{-1}A^{-1}$
- $(AB)^T = B^T A^T$
- $\det(AB) = \det(A)\det(B)$

9. Absolute Value & Inequalities

Absolute Value Properties:

[leftmargin=*]

- $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$
- $|x| \leq a \iff -a \leq x \leq a \quad (a > 0)$
- $|x| \geq a \iff x \geq a$ or $x \leq -a$
- **Triangle Inequality:** $|a + b| \leq |a| + |b|$

Cauchy-Schwarz Inequality:

$$(a_1^2 + a_2^2)(b_1^2 + b_2^2) \geq (a_1 b_1 + a_2 b_2)^2$$