utfcode
utf8.sty 3.10 UTF-8 input encoding 13.06.2000
scanner for code UTF-8 installed.

1. Fundamentals & Hamming Distance

Hamming Distance $d(x, y)$:

$$d(x,y) = \sum_{i=1}^{n} (x_i \oplus y_i) = \text{count of differing bits}$$

Minimum Hamming Distance $d_{\min}$:

$$d_{\min} = \min\{d(c_i, c_j) \mid c_i, c_j \in C, i \neq j\}$$

Error Detection Capability (s bits):

$$s = d_{\min} - 1 \implies d_{\min} = s + 1$$

Error Correction Capability (t bits):

$$t = \lfloor \frac{d_{\min} - 1}{2} \rfloor \implies d_{\min} \geq 2t + 1$$

2. Linear Block Codes (n

Parameters:

- k = Dataword length (bits)
- n = Codeword length (bits)
- $r = n - k$ = Redundancy / Parity bits
- Code Rate $R = \frac{k}{n}$

Generator Matrix $G_{k \times n}$:

$$G = [I_k \mid P] \implies c = d \cdot G$$

where d is dataword $(1 \times k)$ and c is codeword $(1 \times n)$.
Parity Check Matrix $H_{r \times n}$:

$$H = [-P^T \mid I_r] = [P^T \mid I_r] \pmod 2$$

Syndrome Calculation S:

$$S = r_{rec} \cdot H^T$$

- If $S = 0$: No detectable errors.
- If $S \neq 0$: Error detected (S matches error pattern e).

3. Single Error Correcting Hamming Code

Condition for Parity Bits m:

$$2^m \geq k + m + 1 = n + 1$$

where k = data bits, m = parity bits, n = total bits.
Parity Bit Positions: Positions at powers of 2: $1, 2, 4, 8, \ldots, 2^{m-1}$.
Syndrome Decoding:

- Syndrome bits $S = (S_{m-1} \ldots S_0)$ give binary representation of error position.
- If $S = 0$, valid codeword.

4. Cyclic Redundancy Check (CRC)

Polynomial Setup: Let Dataword $D(x)$ of degree $k-1$, Generator $G(x)$ of degree r.
CRC Code Generation:

$$\frac{D(x) \cdot x^r}{G(x)} = Q(x) + \frac{R(x)}{G(x)}$$

Transmitted Codeword $T(x) = D(x) \cdot x^r \oplus R(x)$

CRC Error Detection Capabilities:

- Single-bit errors detected if $G(x)$ has ≥ 2 terms.
- Two isolated errors detected if $G(x)$ does not divide $x^t + 1$ for $t <$ frame length.
- Odd number of errors detected if $(x + 1)$ is a factor of $G(x)$.
- Burst errors of length $L \leq r$ are 100% detected.
- Burst errors of length $L = r + 1$ missed with prob $\frac{1}{2^{r-1}}$.

5. Internet Checksum (1's Complement)

Sender Side: 1. Divide data into 16-bit words. 2. Sum using 1's complement arithmetic (carry-around). 3. Checksum = 1's complement of total sum.
Receiver Side: 1. Sum all words including Checksum. 2. Result should be all 1's (0xFFFF). 3. Complement of result must be 0.

6. Convolutional Codes

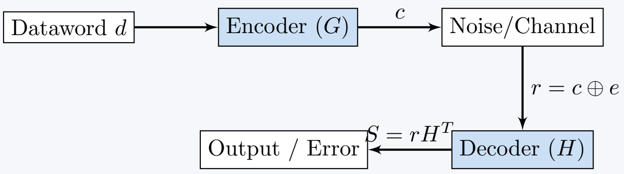
Specification (n, k, K):

- n = output bits, k = input bits
- K = Constraint length (memory units + 1)

Free Distance d_{free}: Determines maximum error capability in sequence decoding:

$$t = \lfloor \frac{d_{\text{free}} - 1}{2} \rfloor$$

7. Error Control Pipeline



8. Quick Comparison Table

Scheme	Overhead	Primary Use
Parity Bit	1 bit	Single bit detection
Hamming Code	$m = \log_2(n+1)$	Single bit correction
CRC	r bits	Burst error detection
Checksum	16 bits	Transport Layer (IP/TCP)