

Advanced Algebra Reference Sheet

Essential Formulas, Laws, and Identities for Rapid Review

1. Exponents and Radicals

$$\begin{aligned} a^m \cdot a^n &= a^{m+n} & \frac{a^m}{a^n} &= a^{m-n} \\ (a^m)^n &= a^{mn} & (ab)^n &= a^n b^n \\ a^{-n} &= \frac{1}{a^n} & a^{1/n} &= \sqrt[n]{a} \end{aligned}$$

2. Factoring Special Polynomials

- **Difference of Squares:**

$$a^2 - b^2 = (a - b)(a + b)$$

- **Sum/Difference of Cubes:**

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

- **Square of Binomial:**

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

3. Quadratic Equations

For the general quadratic equation $ax^2 + bx + c = 0$:

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant (Δ):

$$\Delta = b^2 - 4ac$$

- $\Delta > 0$: Two distinct real roots
- $\Delta = 0$: One repeated real root
- $\Delta < 0$: Two complex conjugate roots

4. Logarithms

Properties for $a, b, x > 0$ and $a \neq 1$:

$$\begin{aligned} \log_a(xy) &= \log_a x + \log_a y \\ \log_a\left(\frac{x}{y}\right) &= \log_a x - \log_a y \\ \log_a(x^r) &= r \log_a x \\ \log_a x &= \frac{\log_b x}{\log_b a} \quad (\text{Change of Base}) \end{aligned}$$

5. Sequences and Series

Arithmetic Sequence:

$$\begin{aligned} a_n &= a_1 + (n - 1)d \\ S_n &= \frac{n}{2}[2a_1 + (n - 1)d] \end{aligned}$$

Geometric Sequence:

$$\begin{aligned} a_n &= a_1 r^{n-1} \\ S_n &= \frac{a_1(1 - r^n)}{1 - r} \quad (r \neq 1) \end{aligned}$$

6. Binomial Theorem

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

where the binomial coefficient is defined as:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$