STATISTICS & PROBABILITY CHEAT SHEET

1. Descriptive Statistics

Measures of Central Tendency:

[leftmargin=*]

- **Sample Mean**: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- **Population Mean**: $\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$
- **Weighted Mean**: $_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$

Measures of Dispersion:

[leftmargin=*]

- **Sample Variance**: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$
- **Population Variance**: $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$
- **Standard Deviation**: $s = \sqrt{s^2}, \quad \sigma = \sqrt{\sigma^2}$
- **Interquartile Range**: $IQR = Q_3 - Q_1$

2. Probability Axioms & Rules

Axioms of Probability:

[leftmargin=*]

1. $0 \leq P(A) \leq 1$
2. $P(\Omega) = 1$
3. $P(A \cup B) = P(A) + P(B)$ if A, B mutually exclusive.

Core Rules:

[leftmargin=*]

- **Complement**: $P(A^c) = 1 - P(A)$
- **Addition Rule**:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- **Conditional Probability**:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad (P(B) > 0)$$

- **Independence**: A, B independent iff:

$$P(A \cap B) = P(A)P(B)$$

3. Total Probability & Bayes' Theorem

Law of Total Probability: For partition $B_1, B_2, \ldots, B_k$ of sample space:

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

Bayes' Theorem:

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$$

4. Random Variables

Expectation & Variance:

[leftmargin=*]

- **Discrete**: $E[X] = \sum xP(X = x)$
- **Continuous**: $E[X] = \int_{-\infty}^{\infty} xf(x)dx$
- **Variance**: $Var(X) = E[X^2] - (E[X])^2$

Linear Properties:

[leftmargin=*]

- $E[aX + b] = aE[X] + b$
- $Var(aX + b) = a^2Var(X)$
- $Var(X \pm Y) = Var(X) + Var(Y) \pm 2Cov(X, Y)$

5. Standard Probability Distributions

Discrete Distributions:

[leftmargin=*]

- **Binomial** $X \sim Bin(n, p)$:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$E[X] = np, \quad Var(X) = np(1-p)$

- **Poisson** $X \sim Pois(\lambda)$:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$E[X] = \lambda, \quad Var(X) = \lambda$

Continuous Distributions:

[leftmargin=*]

- **Uniform** $X \sim U(a, b)$: $f(x) = \frac{1}{b-a}$, $E[X] = \frac{a+b}{2}$
- **Exponential** $X \sim Exp(\lambda)$: $f(x) = \lambda e^{-\lambda x}$, $E[X] = \frac{1}{\lambda}$
- **Normal** $X \sim \mathcal{N}(\mu, \sigma^2)$:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

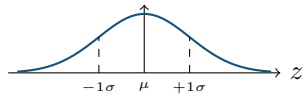
6. Normal Distribution & CLT

Standard Normal Transformation:

$$Z = \frac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$$

Central Limit Theorem (CLT): Let $X_1, X_2, \ldots, X_n$ be i.i.d. with mean μ and variance σ^2. As $n \to \infty$:

$$\bar{X}_n \sim \mathcal{N}(\mu, \frac{\sigma^2}{n}) \implies Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathcal{N}(0, 1)$$



7. Hypothesis Testing Framework

Steps:

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1. State Null (H_0) and Alternative (H_1).
2. Select Significance Level (α).
3. Compute Test Statistic (Z or t).
4. Determine p-value or Critical Region.

Decision Matrix:

	Fail to Reject H_0	**Reject H_0**
H_0 **True**	Correct Decision	Type I Error (α)
H_0 **False**	Type II Error (β)	Correct (Power $= 1 - \beta$)

8. Correlation & Linear Regression

Pearson Correlation Coefficient:

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}} = \frac{Cov(X,Y)}{s_x s_y}$$

Simple Linear Regression:

$$\hat{y} = \beta_0 + \beta_1 x$$

[leftmargin=*]

- **Slope**: $\beta_1 = \frac{Cov(X,Y)}{Var(X)} = r\frac{s_y}{s_x}$
- **Intercept**: $\beta_0 = \bar{y} - \beta_1 \bar{x}$