

1. Number Systems

The study of arithmetic begins with the classification of numbers:

- **Natural Numbers ($\mathbb{N}$):** $\{1, 2, 3, \dots\}$
- **Whole Numbers ($\mathbb{W}$):** $\{0, 1, 2, 3, \dots\}$
- **Integers ($\mathbb{Z}$):** $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- **Rational Numbers ($\mathbb{Q}$):** Numbers of the form $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$.
- **Irrational Numbers ($\mathbb{I}$):** Non-terminating, non-repeating decimals (e.g., $\pi, e, \sqrt{2}$).
- **Real Numbers ($\mathbb{R}$):** Union of rational and irrational numbers.

2. Fundamental Operations

Let $a, b, c \in \mathbb{R}$.

Properties

- **Commutative:** $a + b = b + a$, $ab = ba$
- **Associative:** $(a + b) + c = a + (b + c)$
- **Distributive:** $a(b + c) = ab + ac$

Order of Operations (PEMDAS/BODMAS)

1. Parentheses / Brackets
2. Exponents / Orders
3. Multiplication & Division (Left to Right)
4. Addition & Subtraction (Left to Right)

3. Fractions & Decimals

Operations on Fractions:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

;

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}, \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

4. Divisibility Rules

- **2:** Last digit is even (0, 2, 4, 6, 8).
- **3:** Sum of digits is divisible by 3.
- **4:** Number formed by last two digits is divisible by 4.
- **5:** Last digit is 0 or 5.
- **9:** Sum of digits is divisible by 9.
- **10:** Last digit is 0.

5. HCF and LCM

For any two positive integers a and b :

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

- **HCF (Highest Common Factor / GCD):** Product of the smallest power of each common prime factor in the numbers.
- **LCM (Lowest Common Multiple):** Product of the greatest power of each prime factor involved in the numbers.

6. Percentages

A percentage is a fraction out of 100.

$$\text{Percentage} = \left(\frac{\text{Part}}{\text{Whole}} \right) \times 100\%$$

Percentage Change:

$$\text{Percentage Increase/Decrease} = \left(\frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \right) \times 100\%$$

7. Ratio, Proportion & Variation

- **Ratio:** Comparison of two quantities of the same units ($a : b = \frac{a}{b}$).
- **Proportion:** Equality of two ratios ($a : b = c : d \implies ad = bc$).
- **Direct Variation:** $y \propto x \implies y = kx$
- **Inverse Variation:** $y \propto \frac{1}{x} \implies y = \frac{k}{x}$

8. Profit, Loss & Discount

$$\text{Profit} = \text{Selling Price (SP)} - \text{Cost Price (CP)}$$

$$\text{Loss} = \text{Cost Price (CP)} - \text{Selling Price (SP)}$$

$$\text{Profit \%} = \left(\frac{\text{Profit}}{\text{CP}} \right) \times 100\%$$

$$\text{Loss \%} = \left(\frac{\text{Loss}}{\text{CP}} \right) \times 100\%$$

9. Simple & Compound Interest

- **Simple Interest (SI):**

$$\text{SI} = \frac{P \times R \times T}{100}$$

where P = Principal, R = Rate per annum, T = Time in years.

- **Compound Interest (CI):**

$$A = P \left(1 + \frac{R}{100} \right)^n$$

$$\text{CI} = A - P$$

where A = Amount, n = Number of compounding periods.