Master Class: Foundations of Arithmetic

Comprehensive Lesson Summary & Core Principles

1 Introduction to Number Systems

Arithmetic forms the bedrock of all mathematical disciplines, beginning with the foundational manipulation of numbers. We classify numbers into hierarchical sets that expand from basic counting to complex analytical entities:

- **Natural Numbers ($\mathbb{N}$):** $\{1, 2, 3, \dots\}$ used primarily for counting.
- **Whole Numbers ($\mathbb{W}$):** $\{0, 1, 2, 3, \dots\}$, including additive identity zero.
- **Integers ($\mathbb{Z}$):** $\{\dots, -2, -1, 0, 1, 2, \dots\}$, incorporating negative magnitudes.
- **Rational Numbers ($\mathbb{Q}$):** Numbers expressible as $\frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$.
- **Real Numbers ($\mathbb{R}$):** The set of all rational and irrational numbers residing on the continuous number line.

2 Fundamental Operations and Axioms

Every arithmetic calculation relies on strict structural laws. Let $a, b, c \in \mathbb{R}$. The primary field axioms governing addition and multiplication are:

Property	Addition	Multiplication
Commutativity	$a + b = b + a$	$a \cdot b = b \cdot a$
Associativity	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributivity	$a \cdot (b + c) = a \cdot b + a \cdot c$	
Identity	$a + 0 = a$	$a \cdot 1 = a$
Inverse	$a + (-a) = 0$	$a \cdot a^{-1} = 1 \quad (a \neq 0)$

3 Divisibility, LCM, and GCD

The study of integers heavily depends on division properties. For any two integers a and b (with $b \neq 0$), there exist unique integers q (quotient) and r (remainder) such that:

$$a = bq + r, \quad 0 \leq r < |b|$$

This principle drives the **Euclidean Algorithm** for computing the Greatest Common Divisor (GCD). Furthermore, the relationship between the GCD and the Least Common Multiple (LCM) for any two positive integers a and b is elegantly expressed as:

$$\text{GCD}(a, b) \times \text{LCM}(a, b) = a \cdot b$$

4 Progressions: Arithmetic and Geometric

Sequences model growth patterns encountered across mathematics and finance.

4.1 Arithmetic Progression (AP)

An AP is a sequence where each term after the first differs from the preceding term by a constant common difference d:

$$a_n = a_1 + (n-1)d$$

The sum of the first n terms (S_n) is given by the formula:

$$S_n = \frac{n}{2}\big[2a_1 + (n-1)d\big] = \frac{n}{2}(a_1 + a_n)$$

4.2 Geometric Progression (GP)

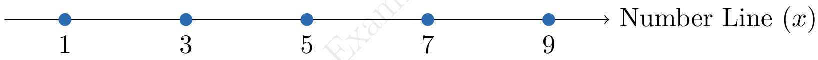
A GP is a sequence where each term after the first is obtained by multiplying the preceding term by a constant common ratio r:

$$a_n = a_1 r^{n-1}$$

The sum of the first n terms when $r \neq 1$ is:

$$S_n = a_1 \frac{r^n - 1}{r - 1}$$

Discrete Arithmetic Progression Step



5 Key Takeaways for Examination

- Always verify order of operations (PEMDAS/BODMAS) before expanding algebraic expressions.
- Use prime factorization trees for rapid LCM and GCD determination.
- Recognize symmetrical properties in word problems involving ratios and proportions.