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Trigonometry: Master Lesson Summary

Core Concepts, Identities, Formulas & Visual Proofs

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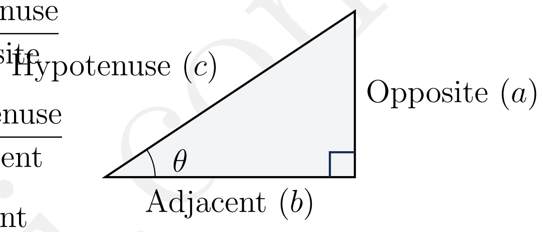
1. Primary Trigonometric Ratios

For a right-angled triangle with an acute angle θ, the fundamental ratios are defined as:

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite}}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\text{Adjacent}}{\text{Opposite}}$$



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2. Fundamental Pythagorean Identities

These identities form the bedrock of trigonometric algebra and hold for all real angles θ:

$$\sin^2\theta + \cos^2\theta = 1 \tag{1}$$

$$1 + \tan^2\theta = \sec^2\theta \tag{2}$$

$$1 + \cot^2\theta = \csc^2\theta \tag{3}$$

3. Angle Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

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4. Double-Angle and Half-Angle Formulas

Double-Angle Formulas:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half-Angle Formulas:

$$\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos(\frac{\theta}{2}) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan(\frac{\theta}{2}) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

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5. Laws for General Triangles

For any arbitrary triangle $\triangle ABC$ with side lengths a, b, c opposite to angles A, B, C:

Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

(where R is the circumradius)

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$