Masterclass Summary: Advanced Arithmetic

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1 Introduction to Number Systems and Fundamental Arithmetic

Arithmetic forms the bedrock of all mathematical reasoning. In this summary, we explore foundational structures, divisibility properties, modular arithmetic, and the formalization of sequences crucial for competitive examinations and rigorous mathematical analysis.

Definition 1.1 (Prime and Composite Numbers)**.** *An integer $p > 1$ is called a **prime number** if its only positive divisors are 1 and p. Integers greater than 1 that are not prime are called **composite numbers**.*

2 The Fundamental Theorem of Arithmetic

Every integer $n > 1$ can be represented uniquely as a product of prime powers, up to the order of the factors.

$$\forall n \in \mathbb{Z}, n > 1 \implies n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k} = \prod_{i=1}^{k} p_i^{a_i}$$

where $p_1 < p_2 < \cdots < p_k$ are distinct primes and $a_i \geq 1$ are positive integers.

3 Progressions and Series

3.1 Arithmetic Progression (AP)

A sequence in which each term after the first is obtained by adding a fixed constant d (the common difference) to the preceding term.

* **General Term (n-th term):**

$$a_n = a_1 + (n-1)d$$

* **Sum of the first n terms (S_n):**

$$S_n = \frac{n}{2}[2a_1 + (n-1)d] = \frac{n}{2}(a_1 + a_n)$$

3.2 Geometric Progression (GP)

A sequence in which each term after the first is obtained by multiplying the preceding term by a non-zero constant r (the common ratio).

* **General Term (n-th term):**

$$a_n = a_1 r^{n-1}$$

* **Sum of the first n terms (S_n):**

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{for } r \neq 1$$

4 Divisibility and Modular Arithmetic

For integers a, b and m (with $m > 0$), we say that a is congruent to b modulo m, written as:

$$a \equiv b \pmod{m}$$

if and only if m divides the difference $a - b$ (i.e., $m \mid (a - b)$).

Example 4.1 (Modular Exponentiation)**.** *To find the remainder when* 3^{100} *is divided by* 7*: By Fermat's Little Theorem, since* 7 *is prime and* $\gcd(3, 7) = 1$*:*

$$3^6 \equiv 1 \pmod{7}$$

Since $100 = 6 \times 16 + 4$*:*

$$3^{100} \equiv (3^6)^{16} \times 3^4 \equiv 1^{16} \times 81 \equiv 81 \equiv 4 \pmod{7}$$

Thus, the remainder is 4*.*

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