Logical Reasoning Cheat Sheet

1. Propositional Logic

Basic Connectives:

- Negation ($\neg p$): NOT p
- Conjunction ($p \wedge q$): p AND q
- Disjunction ($p \vee q$): p OR q
- Implication ($p \to q$): If p, then q
- Biconditional ($p \leftrightarrow q$): p if and only if q

Truth Table Highlights:

p	q	$p \wedge q$	$p \to q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

2. Logical Equivalences

- **Contrapositive:** $p \to q \equiv \neg q \to \neg p$
- **Converse:** $q \to p$
- **Inverse:** $\neg p \to \neg q$
- **De Morgan's Laws:**

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

- **Implication Law:** $p \to q \equiv \neg p \vee q$

3. Syllogisms

Valid standard forms:

- **Modus Ponens:** $p \to q,\ p \vdash q$
- **Modus Tollens:** $p \to q,\ \neg q \vdash \neg p$
- **Hypothetical Syllogism:** $p \to q,\ q \to r \vdash p \to r$
- **Disjunctive Syllogism:** $p \vee q,\ \neg p \vdash q$

4. Logical Fallacies

- **Affirming the Consequent:** $(p \to q \wedge q) \vdash p$ (Invalid)
- **Denying the Antecedent:** $(p \to q \wedge \neg p) \vdash \neg q$ (Invalid)

5. Quantifiers

- **Universal ($\forall$):** "For all" $\forall x P(x)$
- **Existential ($\exists$):** "There exists" $\exists x P(x)$
- **Negation of Quantifiers:**

$$\neg(\forall x P(x)) \equiv \exists x \neg P(x)$$

$$\neg(\exists x P(x)) \equiv \forall x \neg P(x)$$