utfcode
utf8.sty 3.10 UTF-8 input encoding 13.06.2000
scanner for code UTF-8 installed.

1. Exponent and Radical Rules

$$a^m \cdot a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$
$$(a^m)^n = a^{mn} \qquad (ab)^n = a^n b^n$$
$$(\frac{a}{b})^n = \frac{a^n}{b^n} \qquad a^{-n} = \frac{1}{a^n}$$
$$a^0 = 1 \quad (a \neq 0) \qquad a^{1/n} = \sqrt[n]{a}$$
$$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m \qquad \sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$$

2. Algebraic Identities

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a-b)(a+b)$$
$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

3. Quadratic Equations

For a general quadratic equation $ax^2+bx+c=0 \quad (a \neq 0)$:

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant $(D = b^2 - 4ac)$:

- $D > 0$: Two distinct real roots.
- $D = 0$: Two equal real roots $(x = -b/2a)$.
- $D < 0$: Two complex conjugate roots.

Vieta's Formulas (Sum and Product of Roots):

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Vertex of Parabola $y = ax^2 + bx + c$:

$$(h,k) = (-\frac{b}{2a}, c - \frac{b^2}{4a})$$

4. Logarithm Properties

For base $b > 0, b \neq 1$ and $x, y > 0$:

$$\log_b(xy) = \log_b x + \log_b y$$
$$\log_b(\frac{x}{y}) = \log_b x - \log_b y$$
$$\log_b(x^k) = k \log_b x$$
$$\log_b 1 = 0, \quad \log_b b = 1$$
$$\text{Change of Base: } \log_b x = \frac{\log_c x}{\log_c b} = \frac{\ln x}{\ln b}$$
$$b^{\log_b x} = x, \quad \log_b(b^x) = x$$

5. Arithmetic and Geometric Progressions

Arithmetic Progression (AP):

- n-th term: $a_n = a + (n-1)d$
- Sum of n terms: $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$

Geometric Progression (GP):

- n-th term: $a_n = ar^{n-1}$
- Sum of n terms: $S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$
- Infinite GP Sum $(|r| < 1)$: $S_\infty = \frac{a}{1-r}$

Harmonic Progression (HP):

- Terms are reciprocals of an AP sequence.

Means Inequality:

$$\text{AM} \geq \text{GM} \geq \text{HM}$$

$$\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b} \quad (a, b > 0)$$

6. Binomial Theorem

For any positive integer n:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Where the combination coefficient is given by:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

General Term: $T_{k+1} = \binom{n}{k} x^{n-k} y^k$

7. Complex Numbers

Form: $z = a + ib = r(\cos\theta + i\sin\theta) = re^{i\theta}$

$$i = \sqrt{-1}, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$$

Modulus: $|z| = \sqrt{a^2 + b^2}$

Conjugate: $\bar{z} = a - ib$

$$z\bar{z} = |z|^2$$

De Moivre's Theorem: $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

8. Matrices and Determinants

For a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$\det(A) = |A| = ad - bc$$

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad (|A| \neq 0)$$

Cramer's Rule for 2×2 systems:

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D} \quad (D \neq 0)$$