Algebra Ultimate Cheat Sheet

Comprehensive formulas, rules, and properties for rapid review

1. Exponents and Radicals

Laws of Exponents:

$$a^m \cdot a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$
$$(a^m)^n = a^{mn} \qquad (ab)^n = a^n b^n$$
$$a^{-n} = \frac{1}{a^n} \qquad a^0 = 1 \quad (a \neq 0)$$

Radicals:

$$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$
$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} \qquad a^{m/n} = \sqrt[n]{a^m}$$

2. Factoring and Products

Special Products:

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$
$$a^2 - b^2 = (a - b)(a + b)$$
$$(a \pm b)^3 = a^3 \pm 3a^2 b + 3ab^2 \pm b^3$$
$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

Binomial Theorem:

$$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

3. Quadratic Equations

For a quadratic equation of the form:

$$ax^2 + bx + c = 0, \quad a \neq 0$$

The solutions are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant ($\Delta = b^2 - 4ac$):

- $\Delta > 0$: Two distinct real roots.
- $\Delta = 0$: One repeated real root.
- $\Delta < 0$: Two complex conjugate roots.

4. Logarithms

Definition:

$$\log_b(x) = y \iff b^y = x$$

Properties:

$$\log_b(xy) = \log_b(x) + \log_b(y)$$
$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$
$$\log_b(x^k) = k \log_b(x)$$
$$\log_b(1) = 0, \quad \log_b(b) = 1$$

Change of Base Formula:

$$\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$$

5. Complex Numbers

Defined as $z = a + bi$ where $i^2 = -1$.

- **Conjugate:** $\bar{z} = a - bi$
- **Modulus:** $|z| = \sqrt{a^2 + b^2}$
- **Euler's Formula:** $e^{i\theta} = \cos\theta + i\sin\theta$

6. Sequences and Series

Arithmetic Sequence:

$$a_n = a_1 + (n-1)d$$
$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

Geometric Sequence:

$$a_n = a_1 r^{n-1}$$
$$S_n = \frac{a_1(1 - r^n)}{1 - r} \quad (r \neq 1)$$

Infinite Geometric Series ($|r| < 1$):

$$S_\infty = \frac{a_1}{1 - r}$$