1. Standard Form

A quadratic equation in one variable x is an algebraic equation of the second degree, having the general form:

$$\mathbf{ax^2 + bx + c = 0}$$

where $a, b, c \in \mathbb{R}$ and $a \neq 0$.

* **a**: Leading coefficient (cannot be zero). * **b**: Linear coefficient. * **c**: Constant term.

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2. The Quadratic Formula

The solutions (roots) of the standard quadratic equation are given by the famous quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Derived via the method of **completing the square**: 1. Divide by a: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ 2. Shift constant: $x^2 + \frac{b}{a}x = -\frac{c}{a}$ 3. Add $(b/2a)^2$ to both sides.

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3. The Discriminant (Δ)

The expression inside the square root, $\Delta = b^2 - 4ac$, determines the nature of the roots:

* $\boldsymbol{\Delta > 0}$: Two distinct real roots. * $\boldsymbol{\Delta = 0}$: Two equal real roots (one repeated root). * $\boldsymbol{\Delta < 0}$: Two complex conjugate roots ($p \pm iq$).

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4. Roots and Coefficients

Let α and β be the roots of $ax^2 + bx + c = 0$.

* **Sum of Roots:**

$$\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

* **Product of Roots:**

$$\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

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* **Difference of Roots:**

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|}$$

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5. Quadratic Function Graph

The graph of $f(x) = ax^2 + bx + c$ is a **parabola**.

* If $a > 0$, the parabola opens **upward** (minimum value). * If $a < 0$, the parabola opens **downward** (maximum value).

Vertex Coordinates:

$$\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$$

Axis of Symmetry:

$$x = -\frac{b}{2a}$$

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6. Important Identities

* $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ * $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$ * $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ * $\alpha^3 - \beta^3 = (\alpha-\beta)(\alpha^2 + \alpha\beta + \beta^2)$