Calculus: Comprehensive Cheat Sheet

Master formulas for Limits, Derivatives, Integrals, and Series

3

1. Limits Continuity

Definition of Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

—

L'Hôpital's Rule: If $\lim \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$, then:

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

Common Limits: * $\lim_{x\to 0} \frac{\sin x}{x} = 1$ * $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$ * $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = e$

2. Derivatives Rules

Basic Rules: * $\frac{d}{dx}[c] = 0$ * $\frac{d}{dx}[x^n] = nx^{n-1}$ * $(cf)' = cf'$

Product Quotient: * $(fg)' = f'g + fg'$ * $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

3. Derivatives of Trig Functions

* $\frac{d}{dx}(\sin x) = \cos x$ * $\frac{d}{dx}(\cos x) = -\sin x$ * $\frac{d}{dx}(\tan x) = \sec^2 x$ * $\frac{d}{dx}(\csc x) = -\csc x \cot x$ * $\frac{d}{dx}(\sec x) = \sec x \tan x$ * $\frac{d}{dx}(\cot x) = -\csc^2 x$

4. Inverse Trig Exponential

* $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ * $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$ * $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ * $\frac{d}{dx}(e^x) = e^x$ * $\frac{d}{dx}(a^x) = a^x \ln a$ * $\frac{d}{dx}(\ln x) = \frac{1}{x}$

5. Integration Techniques

Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

Substitution:

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

6. Basic Integrals

* $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ * $\int \frac{1}{x}\,dx = \ln|x| + C$ * $\int e^x\,dx = e^x + C$ * $\int a^x\,dx = \frac{a^x}{\ln a} + C$

7. Integrals of Trig Functions

* $\int \sin x\,dx = -\cos x + C$ * $\int \cos x\,dx = \sin x + C$ * $\int \sec^2 x\,dx = \tan x + C$ * $\int \csc^2 x\,dx = -\cot x + C$ * $\int \sec x \tan x\,dx = \sec x + C$ * $\int \tan x\,dx = \ln|secx| + C$

8. Series Expansions

Taylor / Maclaurin Series:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Common Expansions: * $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots$ * $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ * $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ * $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad (|x| < 1)$