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CALCULUS COMPREHENSIVE CHEAT SHEET

1. Limits & Continuity

Definition of Limit:

$$\lim_{x \to a} f(x) = L$$

L'Hôpital's Rule: If $\lim_{x\to a} \frac{f(x)}{g(x)}$ gives $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Standard Limits:

- $\lim_{x\to 0} \frac{\sin x}{x} = 1$
- $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$
- $\lim_{x\to \infty} (1 + \frac{1}{x})^x = e$

2. Differentiation Rules

Fundamental Rules:

- **Power:** $\frac{d}{dx}[x^n] = nx^{n-1}$
- **Product:** $(uv)' = u'v + uv'$
- **Quotient:** $(\frac{u}{v})' = \frac{u'v - uv'}{v^2}$
- **Chain Rule:** $\mathrm{d}_{\overline{dx[f(g(x))]=f'(g(x))g'(x)}}$

Common Derivatives:

- $\frac{d}{dx}[e^x] = e^x, \quad \frac{d}{dx}[a^x] = a^x \ln a$
- $\frac{d}{dx}[\ln x] = \frac{1}{x}$
- $\frac{d}{dx}[\sin x] = \cos x, \quad \frac{d}{dx}[\cos x] = -\sin x$
- $\frac{d}{dx}[\tan x] = \sec^2 x$
- $\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$

3. Integration Techniques

Fundamental Theorems of Calculus: 1. $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ 2. $\int_a^b f(x)\,dx = F(b) - F(a)$
Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

Common Integrals:

- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
- $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int e^x\,dx = e^x + C$
- $\int \sin x\,dx = -\cos x + C$
- $\int \cos x\,dx = \sin x + C$
- $\int \frac{1}{1+x^2}\,dx = \arctan x + C$
- $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$

4. Applications of Calculus

Mean Value Theorem (MVT): If f is cont. on $[a,b]$ and diff. on (a,b):

$$\exists c \in (a,b) \text{ s.t. } f'(c) = \frac{f(b)-f(a)}{b-a}$$

Area Between Curves:

$$A = \int_a^b |f(x) - g(x)|\,dx$$

Volume of Revolution (Disk Method):

$$V = \pi \int_a^b [f(x)]^2\,dx$$

Arc Length:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

5. Sequences & Series

Taylor Series for $f(x)$ at $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Standard Maclaurin Series:

- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots$
- $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \dots$
- $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \dots$
- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad (|x| < 1)$

6. Multivariable Calculus

Gradient Vector:

$$\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})$$

Divergence & Curl:

- div $\mathbf{F} = \nabla \cdot \mathbf{F}$
- curl $\mathbf{F} = \nabla \times \mathbf{F}$

Green's Theorem:

$$\oint_C (P\,dx + Q\,dy) = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dA$$

7. Geometric Visualization

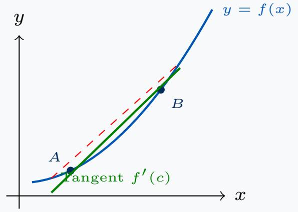

Figure: Secant slope vs. Tangent derivative slope.