Quadratic Equations Master Cheat Sheet

Comprehensive formulas, properties, and advanced problem-solving techniques

1. Standard Form

A quadratic equation in one variable is a second-order polynomial equation of the form:

$$\mathbf{ax^2 + bx + c = 0}$$

where $a, b, c \in \mathbb{R}$ and $\mathbf{a \neq 0}$.

- If $a = 0$, the equation becomes linear ($bx + c = 0$).
- a is the leading coefficient, b is the linear coefficient, and c is the constant term.

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2. The Quadratic Formula

The roots (or solutions) of the standard quadratic equation are given universally by the quadratic formula:

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

This formula is derived by the method of *completing the square.*

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3. Discriminant (Δ)

The expression under the square root, $\Delta = b^2 - 4ac$, determines the nature of the roots:

- $\mathbf{\Delta > 0}$: Two distinct real roots.
- $\mathbf{\Delta = 0}$: Two equal real roots (one repeated root, $x = -b/2a$).
- $\mathbf{\Delta < 0}$: Two complex conjugate roots ($p \pm iq$).

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4. Roots and Coefficients

Let α and β be the roots of $ax^2 + bx + c = 0$. By Vieta's Formulas:

$$\alpha + \beta = -\frac{\mathbf{b}}{\mathbf{a}} \quad \text{(Sum of Roots)}$$

$$\alpha\beta = \frac{\mathbf{c}}{\mathbf{a}} \quad \text{(Product of Roots)}$$

Difference of roots:

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|}$$

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5. Quadratic Expressions

A quadratic polynomial $f(x) = ax^2 + bx + c$ represents a parabola graphically.

- **Vertex Form**: $f(x) = a(x-h)^2 + k$, where the vertex is at $(h, k) = \left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$.
- **Opening**: Opens upward if $a > 0$ (min value at vertex); downward if $a < 0$ (max value at vertex).

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6. Advanced Properties

Forming an Equation from Roots: If α and β are known, the equation is:

$$x^2 - (\text{Sum})x + (\text{Product}) = 0$$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Symmetric Expressions:

- $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
- $\alpha^3 + \beta^3 = (+\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$