

Logical Reasoning: Master Reference

Essential Rules, Fallacies, Syllogisms, and Propositional Logic

1. Propositional Logic

A proposition is a declarative statement that is either True (T) or False (F).

Basic Connectives

Operation	Symbol	Condition
Negation	$\neg p$	True if p is False
Conjunction	$p \wedge q$	True if both are True
Disjunction	$p \vee q$	True if at least one is True
Implication	$p \rightarrow q$	False only if $T \rightarrow F$
Biconditional	$p \leftrightarrow q$	True if $p \equiv q$

Conditional Variations

For implication $p \rightarrow q$: * **Converse:** $q \rightarrow p$ * **Inverse:** $\neg p \rightarrow \neg q$ * **Contrapositive:** $\neg q \rightarrow \neg p$ (Logically equivalent to original)

2. Logical Equivalences

• De Morgan's Laws:

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

• Implication Law:

$$p \rightarrow q \equiv \neg p \vee q$$

• Distributive Laws:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

3. Rules of Inference

Valid argument forms used to construct proofs:

* **Modus Ponens (Law of Detachment):**

$$\frac{p \rightarrow q, \quad p}{\therefore q}$$

* **Modus Tollens (Law of Contraposition):**

$$\frac{p \rightarrow q, \quad \neg q}{\therefore \neg p}$$

* **Hypothetical Syllogism:**

$$\frac{p \rightarrow q, \quad q \rightarrow r}{\therefore p \rightarrow r}$$

* **Disjunctive Syllogism:**

$$\frac{p \vee q, \quad \neg p}{\therefore q}$$

* **Constructive Dilemma:**

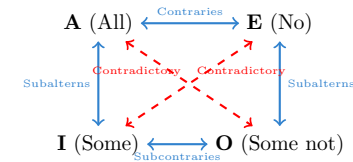
$$\frac{(p \rightarrow q) \wedge (r \rightarrow s), \quad p \vee r}{\therefore q \vee s}$$

4. Syllogisms (Categorical Logic)

Standard forms involving quantifiers (All, Some, No):

- **Universal Affirmative (A):** All S are P .
- **Universal Negative (E):** No S are P .
- **Particular Affirmative (I):** Some S are P .
- **Particular Negative (O):** Some S are not P .

Square of Opposition



5. Logical Fallacies

Errors in reasoning that invalidate an argument.

Formal Fallacies

* **Affirming the Consequent:** $p \rightarrow q, q \vdash p$ (Invalid). * **Denying the Antecedent:** $p \rightarrow q, \neg p \vdash \neg q$ (Invalid).

Informal Fallacies

* **Ad Hominem:** Attacking the arguer instead of the argument. * **Straw Man:** Misrepresenting an argument to make it easier to attack. * **False Dilemma:** Presenting only two options when more exist. * **Circular Reasoning (Begging the Question):** The conclusion is included in the premise. * **Post Hoc Ergo Propter Hoc:** Assuming causation from temporal sequence (A preceded B , therefore A caused B).