# Error Detection and Correction

Data transmitted across communication channels is prone to noise and interference, leading to flipped bits. Error control techniques add redundancy to detect and/or correct errors.

## 1. Fundamental Concepts

- **Hamming Distance ($d_{min}$):** Number of bit positions in which two codewords differ.
- **Error Detection:** To detect up to $s$ errors, we need $d_{min} = s + 1$.
- **Error Correction:** To correct up to $t$ errors, we need $d_{min} = 2t + 1$.

## 2. Block Coding Principles

For a block of $k$ data bits, $r$ redundant/parity bits are added, making a total codeword length $n = k + r$.

$$\text{Code Rate } R = \frac{k}{n}$$

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## 3. Linear Block Codes

A code is linear if the modulo-2 sum of any two valid codewords is also a codeword.

- **Generator Matrix ($G$):** Generates codewords $x = uG$, where $u$ is the message.
- **Parity Check Matrix ($H$):** Satisfies $xH^T = 0$. Dimensions are $(n - k) \times n$.
- **Syndrome ($S$):** Calculated as $s = rH^T$. If $S = 0$, no error detected.

## 4. Cyclic Codes

Linear codes where cyclic shifts of a codeword are also codewords. Represented using generator polynomials $g(x)$ of degree $n - k$.

- **Encoding:** $T(x) = x^{n-k}M(x) + \text{rem}\left[\frac{x^{n-k}M(x)}{g(x)}\right]$
- **CRCs (Cyclic Redundancy Checks):** Widely used in hardware for burst error detection.

## 5. Hamming Codes

Linear error-correcting codes that can correct single-bit errors. For $m$ parity bits, message length $k = 2^m - 1 - m$ and block length $n = 2^m - 1$.

- **Position of Parity Bits:** Powers of 2 $(1, 2, 4, 8, \dots)$.
- **Redundancy Rule:** $d_{min} = 3$, capable of single-bit correction.



## 6. Summary Table

| Scheme | Detect | Correct |
|---|---|---|
| Simple Parity | 1 bit | None |
| 2D Parity | Up to 3 bits | 1 bit |
| Hamming Code | 2 bits | 1 bit |
| CRC | Burst errors | Varies |